

UNIVERSAL DIPOLAR SCATTERING
AND SPIN-ORBIT COUPLING
IN ULTRACOLD GASES OF DYSPROSIUM

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Abstract

Ultracold atomic gases are powerful systems for studying and simulating quantum phenomena, and the magnetic lanthanide atom dysprosium is a promising atom for new areas of quantum simulation owing in part to its large magnetic moment, large spin, and variety of optical transitions.

This thesis presents several studies with ultracold dysprosium. We have characterized both elastic and inelastic scattering properties of dysprosium. In bosonic dysprosium we measured the s-wave scattering lengths via cross-dimensional relaxation and directly imaged the dipolar differential cross section. In fermionic dysprosium we have cooled identical fermions to quantum degeneracy via elastic dipolar collisions. We have also measured inelastic dipolar collision rates in bosonic and fermionic dysprosium with excellent agreement with rates predicted under the first Born approximation, revealing universal scattering behavior. In particular, we demonstrated the fermionic suppression of inelastic dipolar collisions. Finally, we have shown dysprosium to be a promising candidate for fermionic spin-orbit coupling studies due to gas lifetimes as long as 400 ms.

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Chapter 1

Introduction

The Bose-Einstein condensation (BEC) of three alkali atomic species—rubidium, sodium, and lithium—in 1995 proved the power and promise of dilute ultracold atomic gases for quantum experimentation [1–3]. Demonstrations of matter wave inference [4] and atom lasers [5] quickly followed, but efforts also began to make quantum degenerate gases of other isotopes and atoms. In 1998, a BEC of hydrogen was produced [6], and the field took another remarkable step with the realization of the first atomic degenerate Fermi gas (DFG) in 1999 [7]. By 2003, isotopes of all the alkali atoms—save the radioactive francium—had reached quantum degeneracy.

Efforts then pushed beyond hydrogen-like atoms with quantum degenerate gases of ytterbium [8], chromium [9], calcium [10], and strontium [11, 12]. This expansion has culminated in both Bose-Einstein condensates and degenerate Fermi gases of arguably the most complex atoms, dysprosium [13, 14] and erbium [15, 16].

Why the need for so many different species of ultracold atomic gases? While all these gases have generally comparable temperatures and atom numbers, the details crucially differ. Perhaps most clearly, a fermionic isotope (e.g., ^{40}K or ^7Li) cannot

natively undergo Bose-Einstein condensation but instead can form a BEC of Cooper pairs [17].

More subtly, different atoms are better suited for specific applications or quantum simulations. For example, while cesium clocks still serve as the official time and frequency standard, more precise measurements are possible with ultracold strontium in optical lattices [18]. Similarly, dysprosium offers improvements over Rashba and Dresselhaus spin-orbit coupling simulations with fermionic alkali atoms [19, 20] due to dysprosium’s large orbital angular momentum [21] (see Ch. 6).

Finally, new physics can be found unexpectedly when probing a new system. While theorists predicted more Feshbach resonances to be present in dysprosium and erbium compared to the alkalis [22], the extremely dense resonance spectra observed came as a complete surprise [23, 24] (see Ch. 3).

1.1 Properties of dysprosium

Since 2010, dysprosium has proven to be a powerful addition to the list of ultracold atomic species, rich in complexity. Dysprosium is a lanthanide and rare¹ earth element with atomic number 66. A solid metal at room temperature, dysprosium has a melting point of roughly 1400 °C. The five isotopes of dysprosium with relatively high natural abundance are of particular interest for laser cooling and trapping and listed in Table 1.1.

Dysprosium has an electronic configuration $[\text{Xe}]4f^{10}6s^2$. The partially-filled $4f$ -shell gives dysprosium a large orbital angular momentum $L = 6$ and spin $S = 2$ for

¹Rare earth elements are not generally “uncommon.” Instead, they have a rarity in purity as they naturally coalesce, and separating the elements from each other is especially difficult. Rare earth elements include the lanthanide elements along with scandium and yttrium.

isotope	natural abundance	nuclear spin
^{160}Dy	2.3%	0
^{161}Dy	18.9%	5/2
^{162}Dy	25.5%	0
^{163}Dy	24.9%	5/2
^{164}Dy	28.3%	0

Table 1.1: High natural abundance isotopes of dysprosium. Bosons are listed in blue and have nuclear spin of zero. Fermions are listed in red and have nuclear spin of 5/2. The two other stable isotopes, ^{156}Dy and ^{158}Dy , have natural abundances $< 0.1\%$.

a total angular momentum of $J = L + S = 8$ in the ground state. The additional nuclear spin of the fermions $I = 5/2$ yields a hyperfine structure. The lowest hyperfine manifold in ^{161}Dy is $F = J + I = 21/2$; the lowest in ^{163}Dy is $F = J - I = 11/2$. The hyperfine splittings for both ^{161}Dy and ^{163}Dy are shown in Fig. 1.1.

Dysprosium has a large dipole moment of $9.93\mu_B$ with a g_J -factor of 1.24159 for bosonic dysprosium and g_F -factor of 0.946 the fermions. Dysprosium and Terbium are the most magnetic atoms but only ^{158}Tb —a boson—is stable, making ^{161}Dy and ^{163}Dy strictly the most dipolar fermionic atoms.

A multitude of energy levels exists in dysprosium, shown in Fig. 1.1. Two levels² are directly used for the laser cooling and trapping: a 32.2(8)-MHz-wide transition at 421 nm and a 1.78(2) kHz-wide transition at 741 nm [27].

Finally, it is worth noting that dysprosium is highly reactive, which has two effects on the vacuum system. Some materials such as molybdenum—a common material for high-temperature crucibles—react unfavorably and can increase the vacuum pressure; non-reactive alternatives such as tantalum should be used instead. However, dysprosium works as a getter in general, meaning that the vacuum pressure in both

²A third 136(4)-kHz-wide transition at 626 nm is used by the ultracold dysprosium experiment in Stuttgart [26].

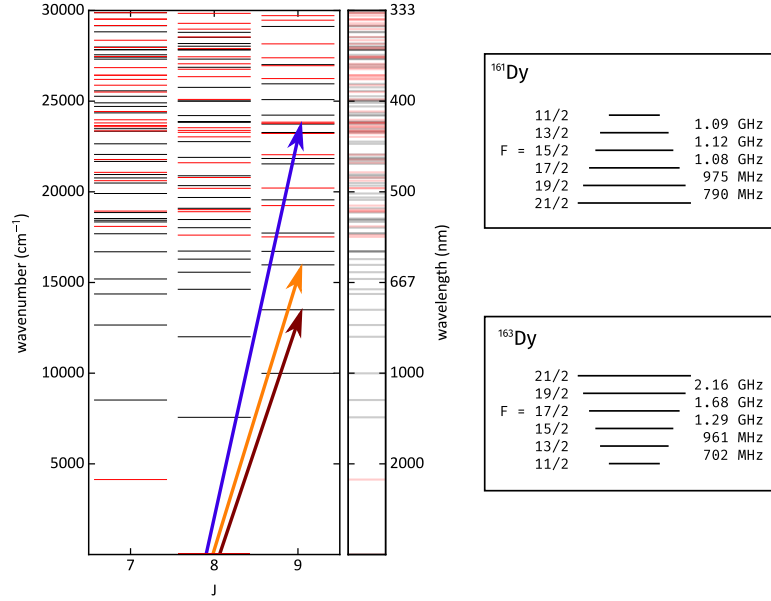


Figure 1.1: Energy levels up to $30,000 \text{ cm}^{-1}$ with $J = 7-9$. States with odd parity are shown in black, and states with even parity are shown in red. An additional presentation that disregards the spread in J is shown on the right side to better convey the energy level density. Three transitions relevant for the laser cooling of dysprosium are indicated on the plot: the 421-nm transition shown in blue, the 626-nm transition shown in orange, and the 741-nm transition shown in red. Data from [25]. Also shown is the hyperfine-splitting of the ground state for ^{161}Dy and ^{163}Dy .

the oven and the main chamber actually improve over time due to the accumulated dysprosium.

The properties of dysprosium outlined here form the basis of the chapters that follow. Chapter 2 presents the methods for making and probing ultracold Bose and Fermi gases of dysprosium. Chapter 3 presents the high density of Feshbach resonances in ultracold dysprosium gases. Elastic collisions—with special attention given to elastic dipolar collisions—are then discussed in Ch. 4. Finally, the primary results of this work on universal inelastic dipolar collisions and spin-orbit coupling in dysprosium gases are presented in Ch. 5 and 6, respectively.

Chapter 2

Quantum degenerate gases of dysprosium

The diversity of ultracold atomic gases is a testament to the broad applicability of the laser cooling and evaporative cooling techniques used to produce the first BECs. Technological improvements such as new laser wavelengths and tighter laser frequency control have certainly been necessary as the field has expanded, but the general principles of engineering favorable photon-atom interactions or evaporative cooling rates remain unchanged. This is especially clear in the production of ultracold dysprosium: Once the specialized lasers are in place, the cooling process is not overly difficult and in some ways is simpler than cooling rubidium.

2.1 Initial trapping and cooling

2.1.1 Magneto-optical trap loading and cooling

Creation of ultracold dysprosium begins by heating solid dysprosium to $\sim 1100^\circ\text{C}$ ¹ in a tantulum crucible. At this temperature a dilute gas forms that reflects the natural abundance of dysprosium's many isotopes. The crucible is pointed at a collimation tube that separates the high-temperature oven from the rest of the trapping chamber, and atoms with appropriately directed velocities pass to the Zeeman slower, forming an atomic beam.

A movable tantulum shield is mounted between the crucible and differential pumping tube such that the atomic beam can be blocked and unblocked. The original design of this atomic shutter involved a rotation mount provided by the oven manufacturer. After the bearings of two rotation mounts failed, a new design shown in Fig. 2.1 was put in place. A steel rod is attached to one end of a flexible bellows via a custom tapped CF vacuum blank. The rod also has a pivot point set by a double-sided CF flange between the bellows and oven chamber. The tantulum flag is placed at the end of the steel rod, and the assembly is actuated by a pneumatic translation stage. After the initial installation the bellows was over-translated and a micro-leak opened over time. After replacing the failed bellows, the actuation was reduced with no incident since.

Once the atomic beam enters the Zeeman slower, a ~ 1 W 421-nm laser beam propagates counter to the atomic beam and selectively slows a single isotope. Details of the Zeeman slower can be found in Refs. [28, 29]. However, a future redesign

¹The melting point of dysprosium is $\sim 1400^\circ\text{C}$.

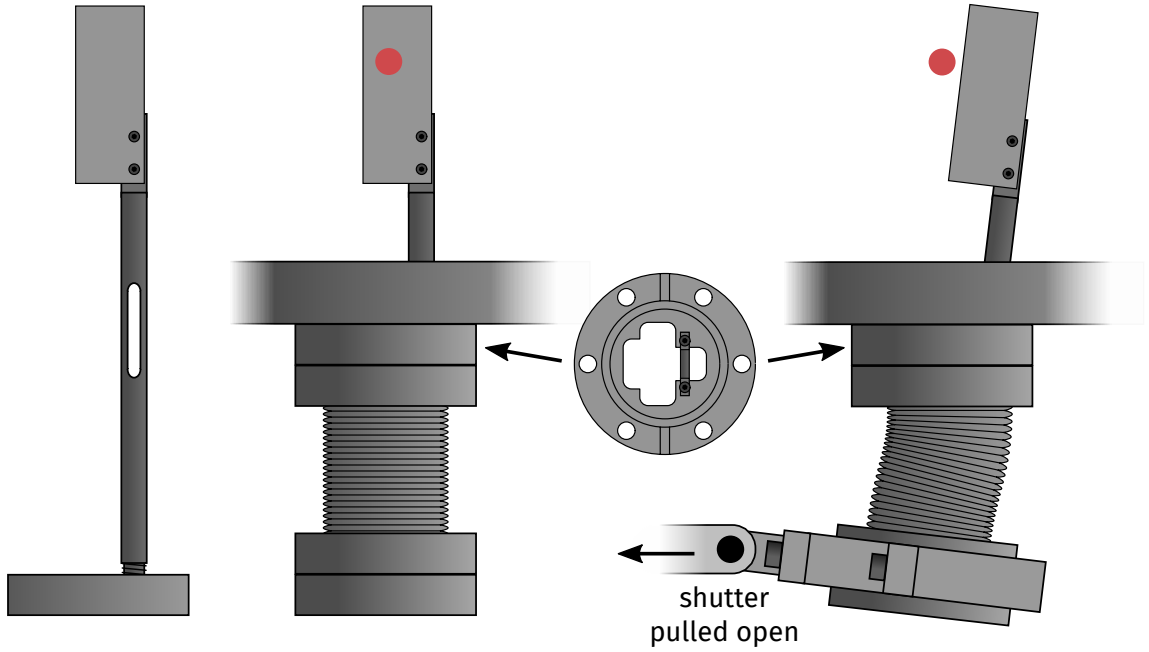


Figure 2.1: A tantulum flag is moved into and out of the atomic beam path (indicated by the red spot) via a pneumatic actuator attached to the flexible bellows. The double-sided CF flange between the bellows and chamber fixes the pivot point of the flag rod.

of this apparatus will do away with the spin-flip Zeeman slower design. The spin-flip Zeeman slower relies on the slowed atoms' inability to adiabatically follow the magnetic field along the slower through a magnetic field zero. Because dysprosium is so magnetic, even a residual transverse magnetic field of a few mG can allow the atoms to follow adiabatically. This severely limits the efficiency of the Zeeman slower, and a non-spin-flip design should have superior performance.

At the end of the Zeeman slower is the main trapping chamber. Three retro-reflected 421-nm beams (making for six beams total) form a 3D repumperless² magneto-optical trap (MOT), detailed in Refs. [30, 31]. More than 1×10^8 atoms at ~ 1 mK are initially trapped in the 421-nm MOT.

The atoms are then transferred to a second MOT formed by five 741-nm, based on a similar trap for erbium [32]. The MOT beams are detuned such that at zero magnetic field, the frequencies would be blue-detuned from the 2-kHz-wide 741-nm transition [13] in dysprosium. However, the atoms are confined away from the quadrupole center, where the beams are optimally red-detuned due to the Zeeman shift. Because the atoms are in a nonzero magnetic field, they are naturally optically pumped into the $m_F = +F$ state by this MOT. More than 1×10^7 atoms at ~ 1 μ K are trapped in the 741 nm MOT. Full details of the 741-nm MOT configuration and operation are found in Ref. [29].

The laser locking scheme for the 741-nm MOT has not been updated since it was detailed in Ref. [29]. However, the PZT that adjusts the cavity length did suffer dielectric breakdown and had to be replaced along with the cavity mirrors. Charring of the PZT from the breakdown coated the cavity mirrors, severely reducing the finesse of the cavity. The cavity spacer was undamaged and could be reused. Both the PZT and mirrors were replaced with identical spares with the one slight modification that we did not allow any epoxy to bridge the two contacts of the PZT in case a breakdown of the epoxy accelerated the breakdown of the PZT. The original performance of the cavity was recovered.

²Dysprosium atoms will occasionally stop interacting with the 421-nm cooling beams by decaying to a metastable “dark” state. However, the atoms can be recovered from the metastable states without repumper beams [30].

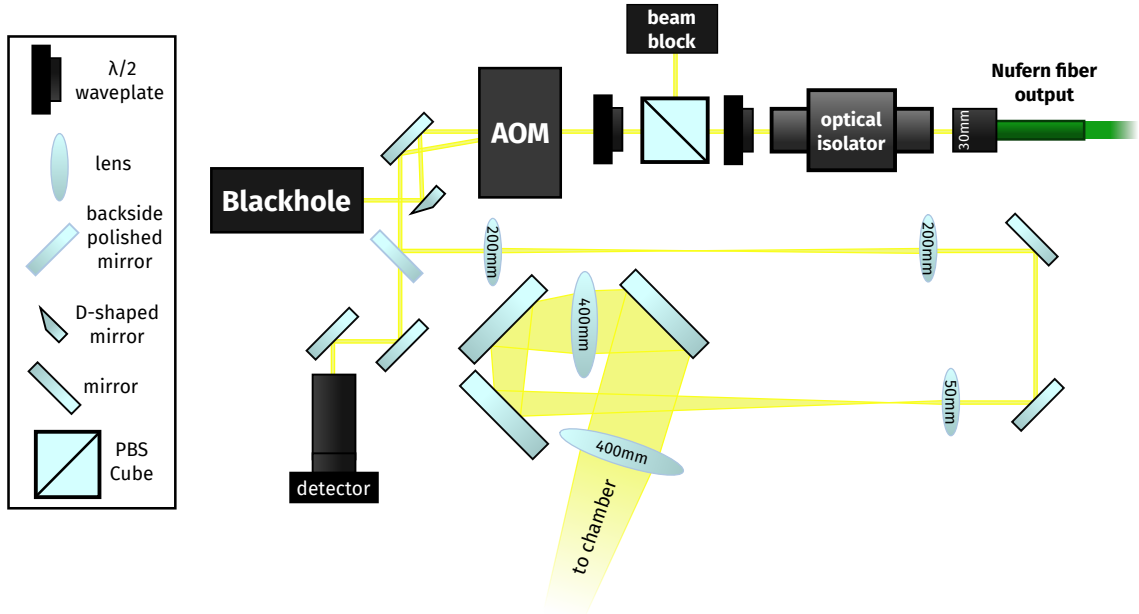


Figure 2.2: The output of a 1064-nm Nufern 50 W amplifier is passed through an AOM. The zeroth-order beam is dumped into a beam block. The first-order beam is intensity stabilized (sampled with a backside polished mirror) and sent onto the atoms. The optics are set such that an angle deviation out of the AOM becomes a horizontal translation on the atoms. By dithering the AOM frequency, the ODT at the atoms can be horizontally broadened.

2.1.2 Optical dipole trapping and evaporation

The last stage of cooling takes place in optical dipole traps (ODTs) formed by 1064-nm laser beams. The atoms are initially loaded into a free-space ODT shown in Fig. 2.2. An acousto-optical modulator (AOM) allows us to servo the optical power (sampled by a backside polished mirror), as well as change the beam aspect ratio by dithering the optical beam horizontally back and forth at ~ 1 MHz. Without dithering, the beam has waists of $24(2) \mu\text{m}$ and $22(2) \mu\text{m}$. Dithering the ODT can expand the trap to an aspect ratio of ~ 5 .

Once in the ODT, the atoms are prepared in the absolute ground state $m_F = -F$

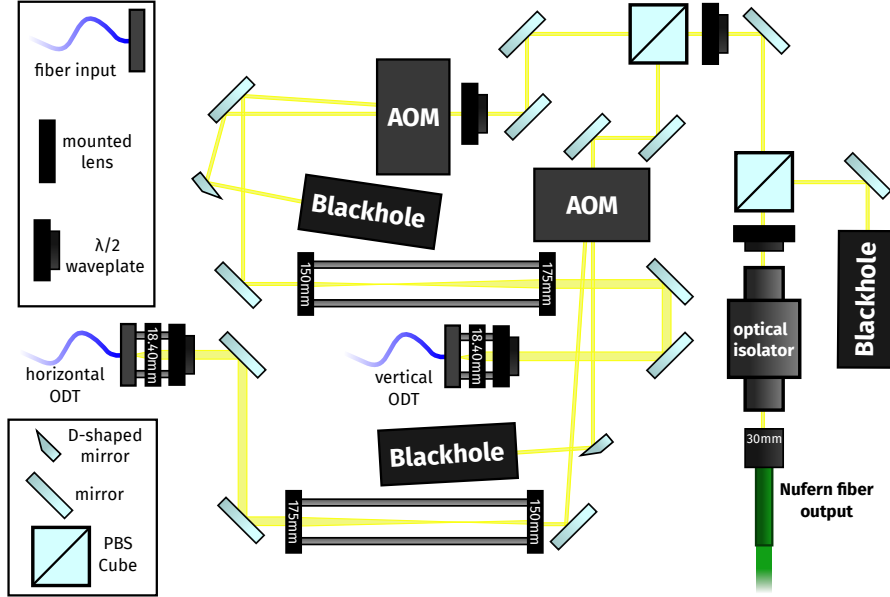


Figure 2.3: The output of a 1064-nm 50 W amplifier is split between two AOMs. The first-order beams are fiber coupled and sent to the chamber to the final horizontal and vertical ODTs.

via rf adiabatic rapid passage. This is necessary due to the high dipolar relaxation rates of dysprosium's metastable states, discussed Ch. 5. The ODT dithering is subsequently ramped off, compressing the cloud for evaporative cooling. Because the ODT has a finite depth, atoms with sufficient energy can escape the trap, and such atoms necessarily carry away more than the average per-particle energy. The remaining atoms' average energy is therefore lowered, cooling the atomic cloud. By lowering the trap depth, we continually remove the hottest atoms from the trap, thus leaving only the coldest, densest atoms behind.

Because efficient evaporation relies on atomic collisions to redistribute energy within the cloud, the peak atomic density is increased by crossing the ODT with a second, vertical ODT. This ODT has a circular waist of $75(2) \mu\text{m}$ and is injected

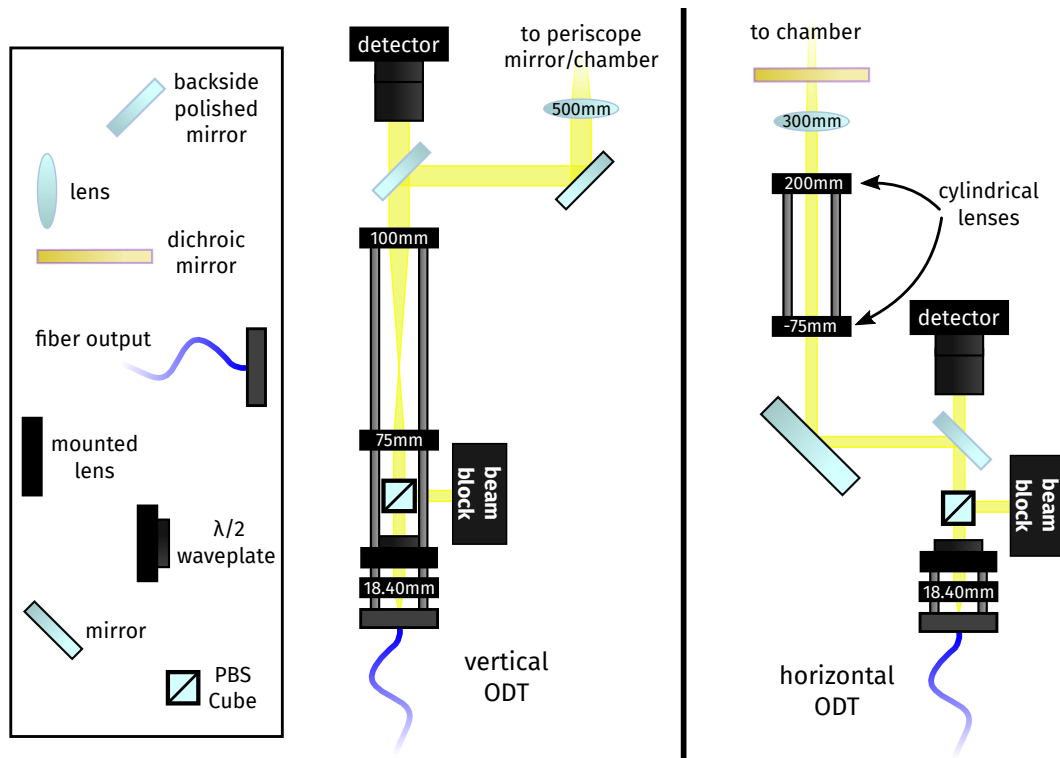


Figure 2.4: The fiber outputs are sampled via backside polished mirrors for intensity stabilization and sent onto the atoms. The cylindrical telescope in the horizontal beam path expand the beam vertically (out of the page) to create a vertically compressed trap at the atoms. This beam also passes through a dichroic mirror before entering the chamber. The mirror transmits 1064-nm light but reflects the 421-nm beam used for imaging the cloud.

into the chamber via an optical fiber to improve pointing stability. The pre- and post-fiber optics of this ODT are shown in Fig. 2.3 and Fig. 2.4, respectively. Initial evaporation takes place in the joint trap formed by these first two ODTs.

The following evaporation sequence applies directly to ^{162}Dy and ^{164}Dy but outlines the basic procedure for ^{160}Dy and ^{161}Dy as well.

The loading ODT begins with a power of 10.0(5) W and reduced to 2.0(1) W in 2 s; at the same time the vertical ODT power is reduced from 4.0(2) W to 3.2(2) W.

The powers are reduced according with the form $P(T) = P(0)/(1 + t/\tau)^\beta$, where τ and β are determined experimentally.

The remaining atoms are transferred to a new trap by simultaneously ramping the powers of the loading ODT, the vertical ODT, and a second horizontal ODT to 0 W, 5.6(3) W, and 2.5(1) W, respectively. The power ramps are linear and take 1 s. The second horizontal ODT is fiber-coupled and has a horizontal waist of 65(2) μm and vertical waist of 35(2) μm . The pre- and post-fiber optics are also shown in Fig. 2.3 and Fig. 2.4, respectively. This trap prevents the cloud from becoming too dense in order to mitigate three-body losses.

With the atoms in the final trap, a BEC is formed by reducing the vertical and horizontal ODT powers to 400(20) mW and 310(20) mW, respectively, are reduced in 5 s. The same functional form as above is used.

2.2 Imaging atomic gases

By far the most common way of measuring ultracold atomic gases is via absorption imaging. When a resonant laser beam passes through the atomic cloud, the atoms remove photons from the beam via absorption and re-emission. By measuring the number of photons removed from the beam, the number of atoms can be determined. Better yet, a 2D spatial profile of the atoms (the 3D distribution of the atoms is integrated along the beams propagation direction) can be determined by measuring the absence of photons with a CCD array.³

Probe beam defects and interferences require that differential images be taken.

³This analysis is valid only when the beam passes through an entirely uniform gas or the local density imaged by each individual CCD pixel is uniform. While the former is generally not true, the latter is often true to good approximation.

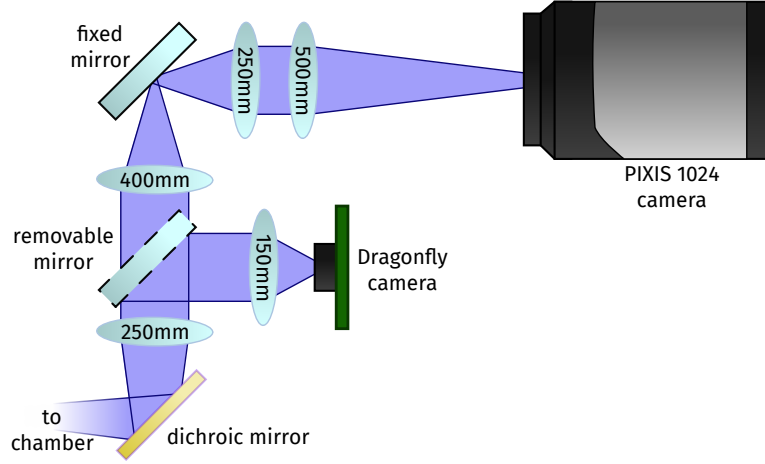


Figure 2.5: The cloud is imaged with a 421-nm probe beam. The beam is reflected by the dichroic mirror that transmits the cylindrical ODT into the chamber. With the removable mirror in place, the image is sent to a Dragonfly camera with a magnification of ~ 0.4 . Otherwise, the image is sent to a PIXIS camera with a magnification of ~ 5 . The beam shown in the figure represents the image (or shadow) carried on the probe.

While it would be convenient to assume an ideal Gaussian profile of the probe beam and attribute any deviations to the presence of atoms, this is wholly unrealistic. Instead, two measurements of the probe beam are made in quick succession: one with atoms (I_{atoms}) and one without (I_{probe}). A third image with the probe beam off accounts for counts on the CCD from other sources (I_{bg}).

Two-level atoms reduce the intensity of the probe beam according to Beer's Law $dI(x, y, z)/dz = -I(x, y, z)\sigma n(x, y, z)$, with the attenuation cross section $\sigma = 3\lambda^2/(2\pi)$ and spatial density of the atoms $n(x, y, z)$. The attenuation of the intensity can be extracted from the three images⁴, and the optical density (OD) of the atomic

⁴A second caveat to this analysis is that the initial intensity of the probe beam must be well below the saturation intensity of the resonant atomic transition. A detailed treatment of high-intensity imaging (rarely applicable for dysprosium) can be found in Ref. [33].

cloud is calculated as

$$\text{OD}(x, y) = -\ln \left[\frac{I_{\text{atoms}}(x, y) - I_{\text{bg}}(x, y)}{I_{\text{probe}}(x, y) - I_{\text{bg}}(x, y)} \right], \quad (2.1)$$

giving the 2D column density of the atomic cloud $\tilde{n}(x, y) = \text{OD}(x, y)/\sigma$.

Because spatial information about the atomic cloud can be obtained through absorption imaging, any property of the cloud that can be mapped on to position can also be obtained. Most importantly, the velocity distribution of a cloud can be measured by time-of-flight imaging: The atoms are released from all trapping potentials and allowed to expand freely. After sufficiently long expansion time, the spatial distribution entirely reflects the velocity distribution rather than the initial trapped distribution.

We use a 421-nm probe beam resonant with the 30-MHz-wide transition in dysprosium for imaging. The two-level atom analysis is valid for atoms in the $m_F = +F$ or $-F$ if probed via σ^+ or σ^- transitions, respectively. For different atomic states or transitions, one must consider optical pumping effects. A full discussion of optical pumping with the 421-nm transition can be found in Ref. [29].

A schematic of the two-magnification imaging system is shown in Fig. 2.5. A dichroic mirror separates the imaging beam from the ODT injected in the same chamber viewport. Lenses form a magnified image of the laser intensity at the atoms on the CCD array. A removable mirror selects between a low-magnification or high-magnification system. Because the optics image the laser intensity at the atoms, the probe beam can be focusing, diverging, or collimated at the atoms: Barring any clipping or aperture effects through the subsequent optics, the density profile and magnification will be the same for all three cases.

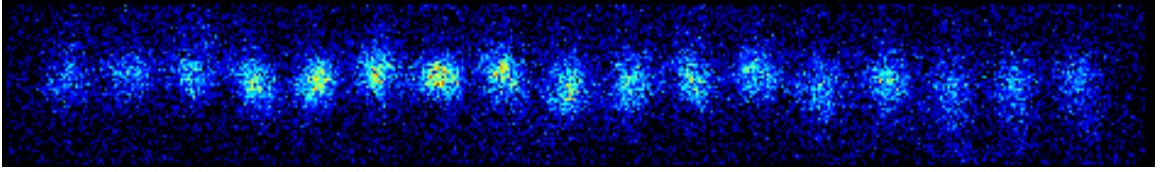


Figure 2.6: Stern-Gerlach image of all 17 ^{162}Dy m_F states. A magnetic field gradient spatially separates the atoms in different m_F states due to the difference in Zeeman energy.

2.2.1 Stern-Gerlach imaging

Just as time-of-flight imaging puts velocity information in the spatial profile of a cloud, Stern-Gerlach imaging encodes spin-state information in the spatial profile. The magnetic Zeeman energy of an atom is determined by—and in the simplest case directly proportional to—the atom’s spin-projection m_F . Thus, a magnetic field gradient will push or pull an atom with a force dependent on the atom’s spin-projection. If the atoms are allowed to freely move under the influence of a sufficiently large gradient, atoms of different spin-states will spatially separate. In this way, the spin content of an atomic cloud can be probed via imaging.

Figure 2.6 shows a cloud with all 17 ^{162}Dy m_F states after being separating them by a Stern-Gerlach measurement. Fluorescence imaging—essentially the opposite of absorption imaging in that the absorbed and re-emitted photons are collected rather than unscattered probe beam photons—was used to image all the spins states at once. The $\sim 200:1$ spread of Clebsch–Gordan coefficients in dysprosium transitions makes absorption imaging of all m_F states difficult; see Ref. [29] for details.

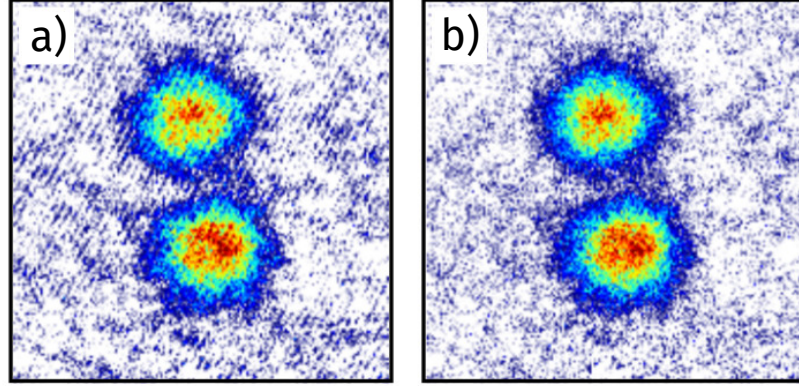


Figure 2.7: (a) An averaged image made from 40 separate absorption measurements. (b) Same image as in (a) after applying the Eigen-image analysis to the individual images, demonstrating significant fringe reduction.

2.2.2 Eigen-image processing

As a final technical detail with regard to absorption imaging, often the final OD images will contain fringes due to slight variations in the probe beam imaged in I_{atoms} and I_{probe} , perhaps due to vibrations or intensity fluctuations. The quality of an absorption image can be improved by constructing an idealized I_{probe} from an appropriate set of reference images. In practice, the I_{probe} images from multiple $(I_{\text{atoms}}, I_{\text{probe}}, I_{\text{bg}})$ sequences form the basis images used.

The method described in Ref. [34] for generating an improved I_{probe} was implemented in a Python package for the Lev Lab. The code is included and discussed in Appendix B. Though the package was designed to work with a custom Lev Lab data format, the implementation is generally useful as a guide.

The “Eigen-image” method is helpful even when other noise suppression techniques are used. For example, random noise can be reduced by simply averaging multiple measurements together. However, Fig. 2.7(a) shows an image that is the average

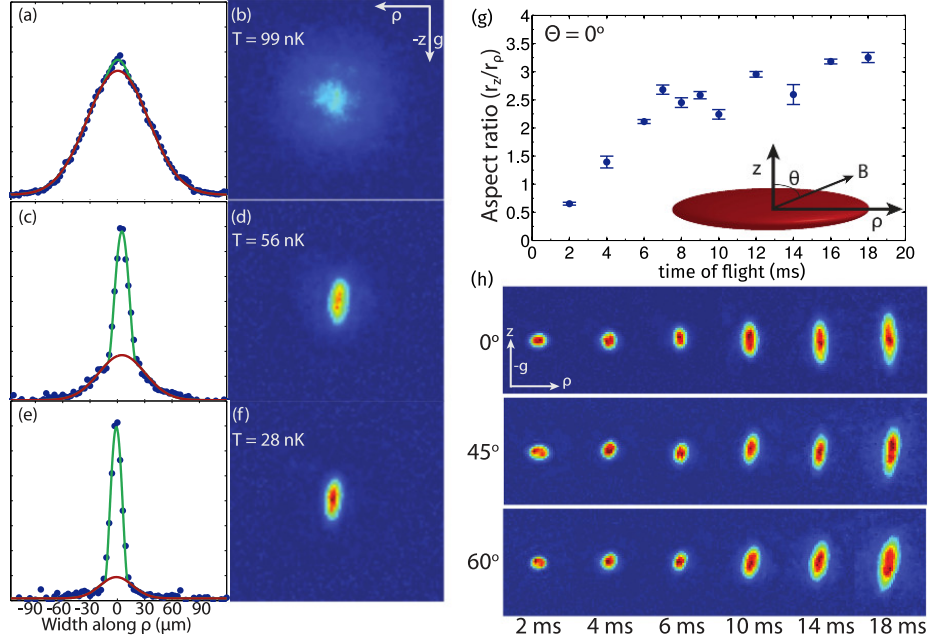


Figure 2.8: (a-f) Time-of-flight profiles of ^{164}Dy showing the emergence of a condensate. (a,c,e) Integrated density profiles with bimodal fits (green) and thermal fraction (red). (b,d,f) Absorption images of the emerging BEC. (g-h) Anisotropic expansion profile versus time after trap release. (g) r_z and r_ρ are the dimensions of the parabolic profile fit to the BEC for $\theta = 0^\circ$. (Inset) Schematic of the oblate trap and magnetic field orientation. (h) Images of the expanding condensate after trap release. The condensate rotates by $7(1)^\circ$ [$9.4(6)^\circ$] with respect to the $\theta = 0^\circ$ expansion orientation for $\theta = 45^\circ$ [$\theta = 60^\circ$]. No BEC forms for $\theta = 90^\circ$.

of 40 different measurements, and fringes persist. After applying the “Eigen-image” analysis before averaging, a clear improvement is made, as shown in Fig. 2.7(b).

2.3 Bose-Einstein condensates

2.3.1 ^{164}Dy BEC

The first quantum degenerate gas of dysprosium was the BEC of ^{164}Dy [27]. The emergence of the BEC is shown in Fig. 2.8. The BEC in Fig. 2.8(e)-(f) has a condensate fraction of 73(4)% with $1.5(2) \times 10^4$ atoms at 28(2) nK. The time-of-flight (TOF) aspect ratio inversion—a hallmark feature of BECs—is shown in Fig. 2.8(g) and the first row of Fig. 2.8(h): The most tightly confined dimension is also the most quickly expanding dimension.

The other two TOF sequences in Fig. 2.8 show the expansion of the cloud with the external magnetic field not oriented normal to the trap geometry (illustrated in the inset of 2.8(g)). The dipole moments of the ^{164}Dy atoms align to the external magnetic field, and the strong dipole-dipole interaction of the atoms causes the atoms to expand in the direction of the field alignment.

The updated ODT configurations (Fig. 2.2, 2.3, and 2.4) and improved evaporation sequence have allowed us to create only slightly larger BECs ($\sim 2 \times 10^4$ atoms)⁵ but in less than half the time (18 s compared to 45 s).

2.3.2 ^{162}Dy BEC

Creating BECs of ^{162}Dy has proven to be much easier than ^{164}Dy due to more favorable scattering properties. Figure 2.9 shows the formation of a BEC at different stages of the evaporation described in Sec. 2.1.2. A nearly pure final condensate of $9.6(4) \times 10^4$ atoms is shown in Fig. 2.9(e) and (f). The BEC has a lifetime of 4.7(1) s

⁵Interestingly, the ^{164}Dy BECs reported from Stuttgart [35, 36] have a similar atom number.

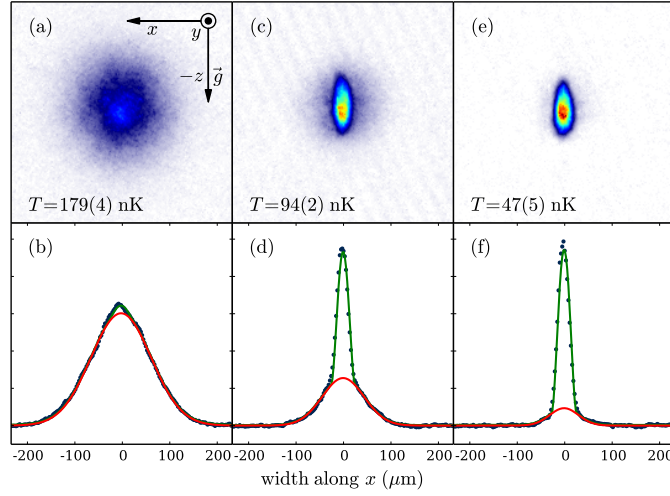


Figure 2.9: Typical single-shot density profiles of ^{162}Dy atomic clouds after 20 ms of free expansion. The top row shows the density distribution, with the colorscale equal for the three pictures. The lower row shows the integrated density along $\hat{x}$ with bimodal fits (green) and thermal fraction (red). (a, b) $3.0(2) \times 10^5$ atoms at the onset of BEC; BEC fraction of 3%. (c, d) Intermediate stage of BEC with $1.5(2) \times 10^5$ atoms; BEC fraction of 40%. (e, f) Nearly pure BEC of $9.6(4) \times 10^4$ ^{162}Dy atoms; BEC fraction of 76%.

due to three-body loss.

2.3.3 ^{160}Dy BEC

Finally, ^{160}Dy can also be condensed but only by modifying the scattering rate via a Feshbach resonance. Most Feshbach resonances in dysprosium are too narrow to control the scattering rate with sufficient precision, but a resonance near 2.8 G provides a 40-mG-wide region centered around 2.717(5) G where we can produce BECs, as shown in Fig. 2.10(b). Outside this region, either the scattering rate is too low [Fig. 2.10(a)] or three-body loss dominates [Fig. 2.10(c)] and no BEC forms.

The condensates of ^{160}Dy are significantly smaller than the other isotopes with

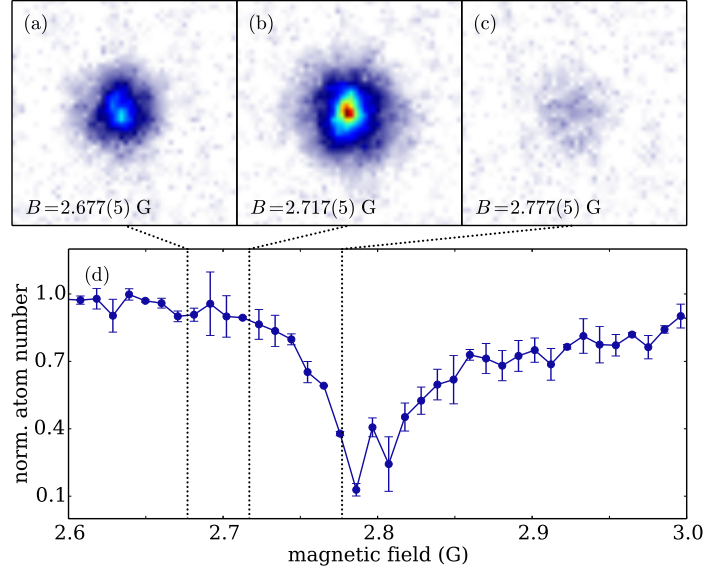


Figure 2.10: Evaporation of ^{160}Dy at near a Feshbach resonance. (a) No BEC forms outside the resonance [$2.677(5)$ G] due to low evaporation efficiency. (b) BECs form in a 40-mG-wide region around $2.717(5)$ G. (c) Three-body losses lead to significant atom loss inside the resonance [$2.777(5)$ G], preventing condensation. (d) Atom-loss spectroscopy of the Feshbach resonance near 2.8 G.

nearly pure condensates of only 800(80) atoms. Furthermore, the evaporation is unstable: The condensate in Fig. 2.11(b) has more atoms but also higher condensate fraction than the condensate in Fig. 2.11(a). This is likely due to the sensitivity of the Feshbach resonance causing different run-to-run evaporation efficiencies and inelastic anti-evaporation (i.e., selectively removing the coldest, densest atoms) [37].

2.4 Degenerate Fermi gases

We produced a degenerate Fermi gas (DFG) of ^{161}Dy about a month before we disassembled the first ultracold dysprosium machine at UIUC and moved to Stanford. Details of the DFG are reported in Ref. [14]. Although ^{163}Dy is more abundant than

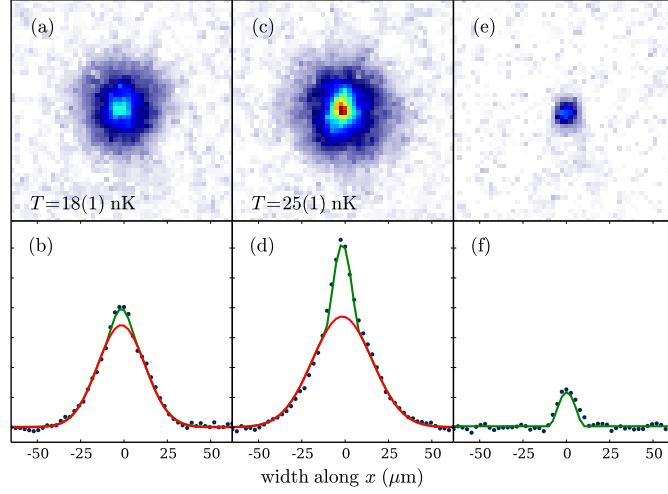


Figure 2.11: Density profiles of ^{160}Dy atomic clouds. The bottom row includes bi-modal fits (green) and thermal fraction (red). (a, b) $7.6(8) \times 10^3$ atoms at the onset of BEC. $T = 18(1)$ nK and BEC fraction is approximately 5%. (c, d) Intermediate stage of BEC with $1.1(1) \times 10^4$ atoms at $T = 25(1)$ nK. BEC fraction is approximately 14%. (e, f) Nearly pure BEC of $8.0(8) \times 10^2$ ^{160}Dy atoms.

^{161}Dy , the former exhibits drastic losses due to hyperfine changing collisions⁶. The negative nuclear magnetic moment of ^{161}Dy means ^{161}Dy is naturally prepared in the lowest hyperfine manifold in the initial MOT stage.

This first DFG was produced via sympathetic cooling with ^{162}Dy , which requires co-trapping of both isotopes. We only have a single 421-nm cooling laser system, so ^{161}Dy was first slowed, cooled, and finally loaded into a magnetic trap. The lasers were then tuned to slow and cool ^{162}Dy .

Two 741-nm lasers then simultaneously cooled ^{161}Dy and ^{162}Dy in the second MOT stage. After further cooling, both isotopes were loaded into the ODT and prepared in the $F = -F$ state for forced evaporation. The two isotopes remain in

⁶Additional preparation into the lowest hyperfine manifold via either microwave or optical transitions should enable DFGs of ^{163}Dy in the future.

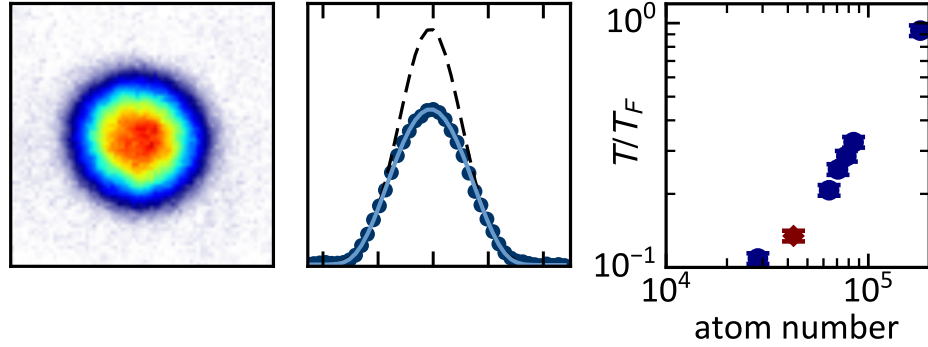


Figure 2.12: DFG of ^{161}Dy with $4.3(2) \times 10^4$ atoms at $T/T_F = 0.125(4)$ produced via single-species evaporation. Collisions between identical fermions are mediated by the dipole-dipole interaction. (a) Six image average of the DFG after time of flight. (b) Momentum distribution with Thomas-Fermi fit (solid) and Maxwell-Boltzmann fit (dashed) showing the reduction in low-momentum states due to the Pauli exclusion principle. (c) Single-species evaporation trajectory of ^{161}Dy . The red diamond marks the cloud shown in (a) and (b).

good thermal contact throughout evaporation, and a final DFG with $6.0(6) \times 10^3$ atoms and $T/T_F = 0.21(5)$ was obtained.

We also demonstrated the evaporation of identical fermions to $T/T_F = 0.7$ in Ref. [14], whereas identical fermions typically stop thermalizing with $T/T_F \gg 1$ due to suppressed scattering below the p -wave threshold. Elastic dipolar collisions are not suppressed, however, and DFGs of both ^{161}Dy and ^{167}Er [16] have been produced via dipolar scattering.

With the improved magnetic field stability and new ODT configurations we can now produce ^{161}Dy DFGs with $4.3(2) \times 10^4$ atoms at $T/T_F = 0.125(4)$ without sympathetic cooling as shown in Fig. 2.12.

2.5 Summary

The laser cooling and trapping of dysprosium has progressed such that we can now produce ^{161}Dy DFGs with 4×10^4 atoms at $T/T_F = 0.12$ via single-species evaporation and ^{162}Dy BECs of $\sim 10^5$ atoms. Condensates of ^{164}Dy and ^{160}Dy have been produced but with lower atom number. DFGs of ^{163}Dy would require additional preparation into the lowest hyperfine manifold.

Chapter 3

Feshbach resonances

The simplest and most common atom-atom interaction in ultracold experiments is *s*-wave scattering¹, which results from the short-ranged van der Waals interaction. The details of *s*-wave interaction can be captured by a single length scale: the scattering length, a . While the scattering length varies from atom to atom and isotope to isotope due to microscopic details, $100a_0$ (where a_0 is the Bohr radius) is the typical length scale.

Feshbach resonances provide a handle to change the scattering length of an atom, allowing the experimentalist to adjust the interaction strength of a systems [38]. Feshbach resonances occur when a molecular bound state of two atoms is energetically resonant with the atoms' collisional energy. This resonance causes a mixing between the closed channel—i.e., the molecular potential that is inaccessible to the atoms at large interatomic distances but supports the bound state—and the open channel on which the atoms approach. This mixing of the two channels causes the scattering

¹In fact, the term “ultracold” refers to the temperature of the system being lower than the *p*-wave centrifugal barrier. Thus, *s*-wave—having no centrifugal barrier—is the only energetically favorable partial wave for collisions.

length a to diverge.

Because the closed and open channels typically have different magnetic moments, a common way to bring the bound and collisional energies into resonance is by tuning a magnetic field. In this way, this scattering length near a magnetic Feshbach resonances at field B_0 has the form

$$a(B) = a_{bg} \left(1 - \frac{\Delta}{B - B_0} \right), \quad (3.1)$$

where B is the magnetic field, a_{bg} is the resonance-free scattering length, and Δ is the width of the resonance.

Alkali atoms exhibit Feshbach resonance densities of roughly 1 resonance per 100 G. Such resonances have widths on the order of 10 G. However, the density of resonances in the lanthanide atoms dysprosium and erbium is greatly increased. While an increased density was expected [22], the actual density far exceeds the predicted value.

3.1 Low-field feshbach resonances in dysprosium

We first reported the high density of Feshbach resonances in ultracold dysprosium in Ref. [23]. We used a set of coils in near-Helmholtz configuration providing a field of ~ 2 G/A, where the current is set and stabilized with an analog feedback circuit allowing us to apply a field of any value within the range of 0-6 G along gravity. We have calibrated the field and verified its stability over several days by rf spectroscopy: Transitions between magnetic sublevels are driven by a weak rf field, and the atoms eventually release their Zeeman energy as kinetic energy [39, 40]. This energy is larger

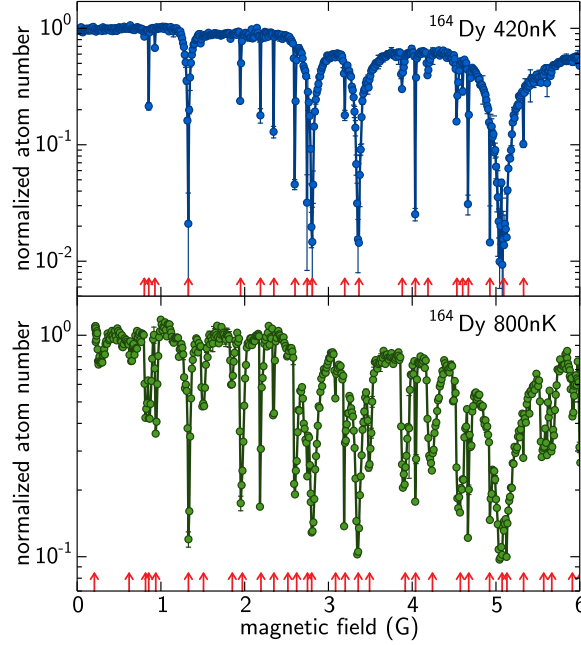


Figure 3.1: Atom-loss spectrum for bosonic ^{164}Dy . The top panel displays data taken with a $\sim 420\text{ nK}$ sample. The lower panel shows a measurement of the same isotope ^{164}Dy at higher temperature $\sim 800\text{ nK}$. New resonances appear in the warmer sample. The arrows show the position of the resonances obtained by fitting a Gaussian function to each individual feature.

than the depth of the ODT and the atoms are lost from the trap. We mapped the resonant loss feature at fixed magnetic field by changing the frequency of the rf field, and a fit to the data determined the position of the resonance to within better than 1 mG. Repeating this calibration across many days revealed a field offset drift of less and 5 mG per week.

In order to map the spectrum of Feshbach resonances as a function of magnetic field, we quickly ($<10\text{ ms}$) ramped the magnetic field to the desired target value after the initial preparation sequence. The power of the ODT beams was then dropped by $\sim 30\%$, leading to an immediate loss of atoms and an out-of-equilibrium velocity distribution of the atomic ensemble. Close to a peak of a Feshbach resonance trap

loss from the non-adiabatic lowering of trap depth is accompanied by trap loss from increased inelastic three-body collisions [38].

The center and width of Feshbach resonances were identified by measuring the temperature and atom number after a fixed holding time. Measurements were performed via absorption imaging with a 10-ms time-of-flight. In order to ensure comparable conditions among datasets, we adjusted the loading, cooling, and probing sequence slightly for each individual isotope. The final trap geometry, holding time, atomic density, and temperature for each isotope is reported in the following.

The data for the bosonic isotope ^{164}Dy is in Fig. 3.1. The top panel shows the normalized atom number as a function of magnetic field after the drop in ODT trap depth. Final trap frequencies were [266, 76, 299] Hz (x - y plane and $\hat{z}$, respectively, where gravity points along $\hat{z}$) and the trap was held for 375 ms. For every field value, we repeated the measurement twice and show the mean of those experimental runs with the error bars given by the absolute mean deviation. The data was taken in blocks of 250 mG in 10 mG steps. We repeatedly measured the atom number at a reference magnetic field, which is the same for all blocks, to normalize the atom number. For the data presented in Fig. 3.1 (top panel), we used 0.485 G as a reference field, where we measured 3×10^5 atoms at an average density of $9 \times 10^{13} \text{ cm}^{-3}$ and a temperature of 420 nK. Many more resonances (21 below 6 G or $\sim 3/\text{G}$) were observed in ^{164}Dy than in collisions of alkali atoms. The arrows in the figure show the position of the resonances obtained by fitting a Gaussian function to each individual resonance. Many features show a very small width, ~ 10 mG.

We repeated the measurement with ^{164}Dy at a higher temperature, shown in the lower panel of Fig. 3.1. The final trap frequencies of the ODT are [291, 85, 328] Hz,

and the ODT is held for 100 ms. At the reference magnetic field of 0.535 G, we measure 4.4×10^5 atoms at an average density of $8 \times 10^{13} \text{ cm}^{-3}$ and a temperature of 800 nK. The spectrum is qualitatively similar to the colder gas; however, all resonances are broadened and new resonance features are apparent. The total number of resonances observed is 32, resulting in a density of ~ 5 resonances per gauss.

The broadening with increased temperature suggests contributions of higher-order partial waves to the collisional process. Indeed, theoretical calculations require the contributions of many partial waves to achieve numerical convergence due to the anisotropy of the dipolar interaction [22, 41]. The sample temperature affects the particular contribution weight, resulting in a narrowing and vanishing of resonances. This is in contrast to alkali-metal atoms in which the typical p -wave barrier is in the mid- to high- μK regime, below which higher partial-wave resonances cannot be observed and the spectrum no longer changes as a function of temperature [38].

In general, one expects the spectrum of Feshbach resonances to be highly dependent on isotope [22], and we repeated the above measurements with ^{160}Dy and ^{162}Dy at very low but non-condensed temperatures. The scanned magnetic field range, resolution, number of experimental runs, and randomization was chosen to be identical to the previous measurement. Figure 3.2 (top panel) shows the resulting spectrum for the bosonic isotope ^{160}Dy : trap frequencies [259, 53, 287] Hz held for 375 ms with 1.4×10^5 atoms, $1 \times 10^{13} \text{ cm}^{-3}$ density, and a temperature of 350 nK. The lower panel shows the ^{162}Dy data: trap frequencies [251, 7, 276] Hz held for 300 ms with 4.9×10^4 atoms, $9.3 \times 10^{11} \text{ cm}^{-3}$ density, and a temperature of 500 nK. Both spectra are qualitatively very similar to ^{164}Dy (Fig. 3.1) and show 11 (9) resonances for ^{160}Dy (^{162}Dy), though neither exhibits the relatively wide resonance of ^{164}Dy within the 0-6

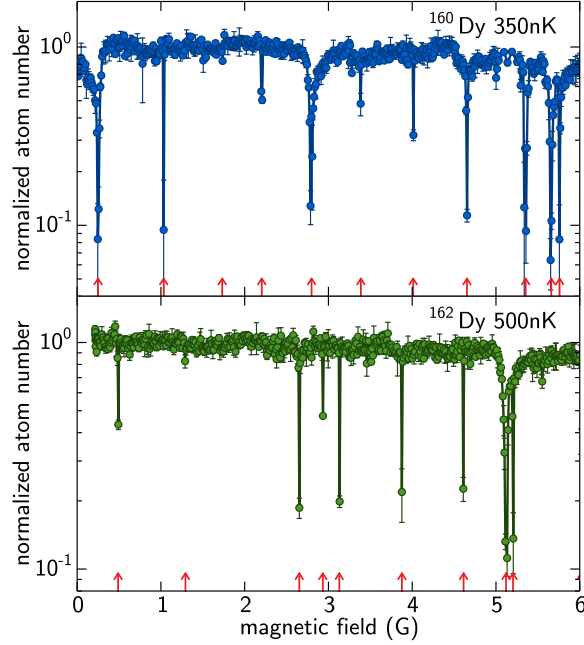


Figure 3.2: Atom loss spectra as a function of magnetic field for the bosonic isotopes ^{160}Dy (top) and ^{162}Dy (bottom). The spectra lack the broad feature observed in ^{164}Dy (Fig. 3.1), but are otherwise qualitatively similar. The arrows show the position of the resonances obtained by fitting a Gaussian function to each individual feature.

G scan range.

Figure 3.3 shows data taken with the fermionic isotope ^{161}Dy . The top panel shows a measurement over the full range of 6 G with a resolution of 7.8 mG, trap frequencies of [326, 11, 360] Hz, an ODT hold time of 240 ms, and 4.9×10^4 atoms at an average density of $2 \times 10^{12} \text{ cm}^{-3}$ and a temperature of 510 nK. The temperature in this configuration is a factor of 1.5 higher than the Fermi temperature. The spectrum is much denser than the bosonic spectra in Figs. 3.1 and 3.2: We observe a very large number of ultra-narrow features and the absence of broad resonances. The lower panel of Fig. 3.3 shows a 0.6 G sweep with an increased resolution of 1.3 mG. The two datasets taken on two consecutive days show excellent agreement and reveal a width

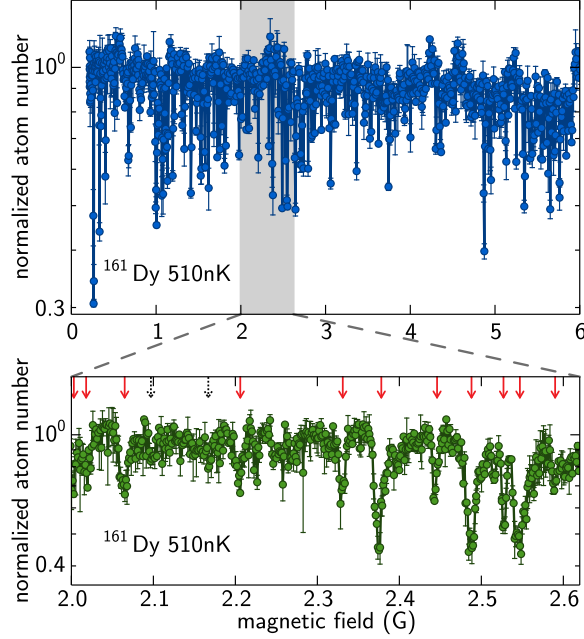


Figure 3.3: Atomic loss spectrum for the fermionic isotope ^{161}Dy . The top panel shows a full scan over 6 G whereas the lower panel shows a close-up with much higher resolution. As is apparent from the close-up plot, the ultra-narrow features in the top panel are scattering features. The arrows in the lower panel show the positions of the resonances deduced from the upper panel; the arrows in black (dashed) mark false resonances, i.e., statistical fluctuations in the upper panel not reproduced in the lower panel with high-resolution data.

to features that appear only as a single data point in the lower resolution, wide-scan data.

We observe more than 70 resonances below 6 G for fermionic ^{161}Dy , yielding a density that exceeds 11 resonances per gauss. The larger density of resonances in fermionic compared to bosonic dysprosium can qualitatively be explained by the nonzero nuclear spin of the fermionic ^{161}Dy , which is zero for the bosonic isotopes. The resulting hyperfine coupling provides an additional spin degree of freedom, greatly increasing the number of available collisional channels.

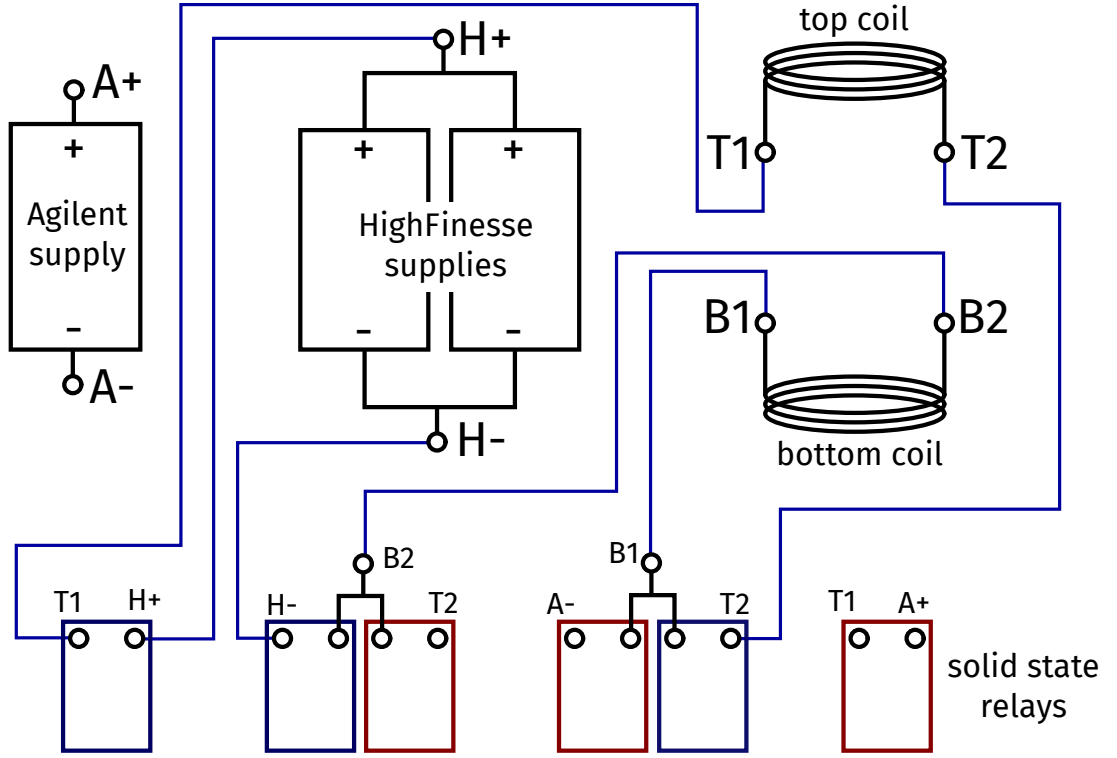


Figure 3.4: Current control for generating 0–40 G magnetic fields. Six solid state relays (red and blue rectangles) control the current direction and source and are open/closed via TTL. With the blue relays closed and red relays open, the two High-Finesse 3 A (total 6 A) supplies are connected to the coils in a Helmholtz configuration. With the red relays closed (connections not shown) and blue open, the Agilent 160 A supply is connected to coils in an anti-Helmholtz configuration for the MOT field.

3.2 High-field Feshbach resonances

In order to extend the measurement of Feshbach resonances to fields above 6 G we used the coil pair originally used for the MOT magnetic gradient. This required a switching circuit that could select between a Helmholtz and anti-Helmholtz configuration. Moreover, the Agilent power supply and home-built current servo would not provide sufficient field stability to resolve the narrowest resonances, so an alternate

power supply was also needed.

The solution to both problems is shown in Fig. 3.4. Six solid-state relays are used to switch between the MOT gradient from the Agilent supply and the bias field from two HighFinesse supplies. The HighFinesse supplies in parallel can generate a magnetic field from 0–40 G (with 0–8 A) but would not be able to produce a sufficiently large magnetic gradient for the MOT which requires ~ 30 A of current. Connected terminals in Fig. 3.4 share labels (e.g., H+ or T2). While the connections for the Helmholtz configuration are shown by the blue traces, the anti-Helmholtz connections are omitted for clarity.

The HighFinesse supplies have an internal servo that accepts an analog control voltage to set the output current. However, using a 16-bit DAC to control 0–40 G limits the field resolution to ~ 1.2 mG (only 15 bits are available for the output range of 0–10 V). To improve the field resolution, we divide down the DAC output by a factor of ~ 20 with a simple resistive voltage divider. This reduces the dynamic range while increasing the resolution.

The HighFinesse supplies also have a DC “manual” offset that can be set by a dial on the front panel and controlled by a 5 V trigger voltage. By using both the high resolution analog control and the manual offset, we can scan the magnetic field with sub-100 μ G resolution up to 40 G.

The atom-loss spectra in Fig. 3.5 cover 0–38 G with 20 mG resolution. The spectra were measured by decompressing the cloud (i.e., lowering the ODT trap frequencies) in 50 ms and then moving to the desired field while recompressing the trap. The gas was then held for a fixed amount of time to let the atom loss reflect the modified scattering length.

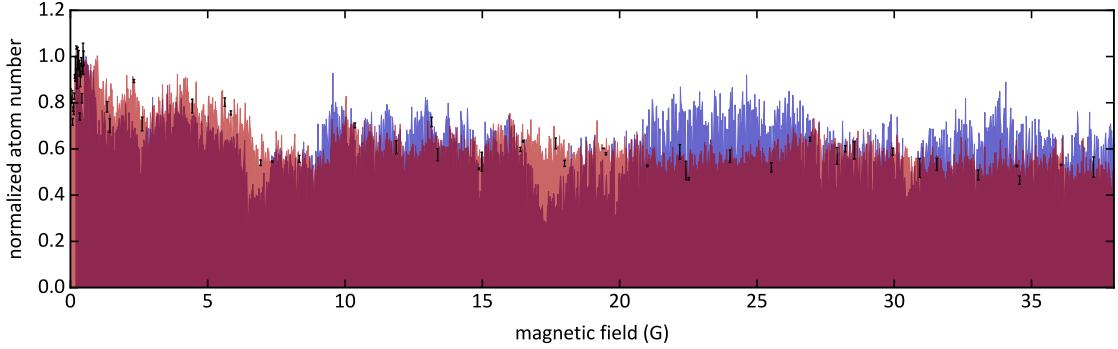


Figure 3.5: Normalized atom loss spectroscopy of ^{161}Dy in $m_F = -21/2$ at $8\ \mu\text{K}$ (red spectrum) and $500\ \text{nK}$ (blue spectrum). A stepsize of $\sim 20\ \text{mG}$ was used over the $0\text{--}38\ \text{G}$ range.

One difficulty in measuring a spectrum over such a large range of magnetic fields is that the initial atom number may decrease at larger magnetic fields because more resonances are crossed during the magnetic field sweep. To eliminate this effect, the sweeps were configured to cross roughly the same number of resonance regardless of the final field. Two example ramps are illustrated in Fig. 3.6. The field is increased at a rate $+\Delta B$ for time t_u and then decreased at a rate $-\Delta B$ for time t_d such that $t_u + t_d$ is a constant (20 ms). The final field can be changed by adjusting t_u and t_d together. Using this method, the initial atom number was the same across the entire $0\text{--}38\ \text{G}$ region².

The spectrum presented in red in Fig. 3.5 was taken at $8\ \mu\text{K}$, and the other in blue was taken at $500\ \text{nK}$ as in Fig. 3.3. While the resolution is much too low to actually resolve resonances, broad features are apparent, especially in the low-temperature data.

²One other subtlety is that larger fields could affect atom counting due to eddy currents persisting during imaging. This effect is easily mitigated by either increasing the time-of-flight or taking an atom number calibration measurement

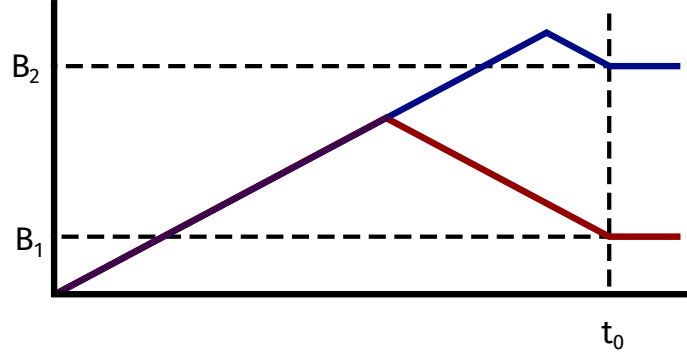


Figure 3.6: Examples of the magnetic field sweeps used for the 0–38 G atom loss spectroscopy. The sweep and sweep rate are constant for all final fields. By changing the sign of the sweep rate part way through the ramp, a different final magnetic field is obtained. Such ramps cross roughly the same number of resonances regardless of final magnetic field.

The rise and fall of the low-temperature data is largely matched in the high-temperature data up to about 7 G. Above this field the landscape in the high-temperature data flattens significantly. One possible explanation for this lower contrast is the presence of more resonances at higher fields: More resonances could have an averaging effect on the spectrum, suppressing the variation.

This explanation is supported by the high-resolution spectra taken around 34 G shown in Fig. 3.7. The spectra presented in blue was taken at 500 nK with the atoms prepared in $m_F = -21/2$ with a resolution of ~ 0.3 mG. The spectrum is ultradense with 20 resonances in a region of 275 mG. This comes out to a density of more than 70 resonances per Gauss, $6\times$ denser than the 0–6 G spectrum. Even with a resolution of $300\ \mu\text{G}$ some resonances are not fully resolved and even more resonances may exist.

This $m_F = -21/2$ spectrum in Fig. 3.7 was taken by first decompressing the gas in 100 ms, subsequently jumping the field to the final value, and finally recompressing the gas over 100 ms while eddy currents damp. The gas was held in the recompressed

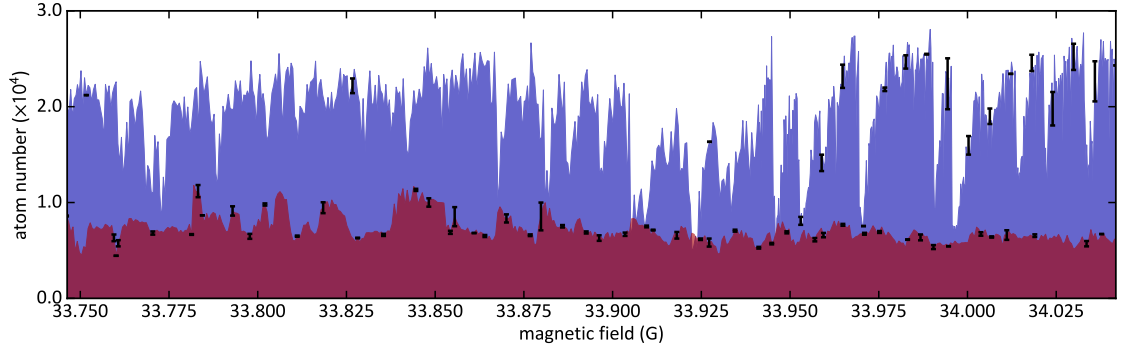


Figure 3.7: Atom loss spectroscopy of ^{161}Dy at 500 nK in $m_F = -21/2$ with $\sim 300 \mu\text{G}$ resolution (blue spectrum) and $m_F = -19/2$ with $\sim 600 \mu\text{G}$ resolution (red spectrum).

trap for 60 ms before imaging.

A similar procedure was used for the $m_F = -19/2$ spectrum in Fig. 3.7 with modifications for the spin preparation. After decompressing, the field was jumped to 33.967(5) G (a low-loss field for $m_F = -21/2$) and eddy currents were allowed to damp. The atoms were then transferred to $m_F = -19/2$ via rf ARP³. The field was then jumped to the final value, and the cloud recompressed in 100 ms and held for 60 ms.

The atom loss with the atoms prepared in $m_F = -19/2$ is much greater than the $m_F = -21/2$ loss. Features are still clear but broader and with much less contrast. This may indicate an even higher density of resonances due to the increased number of open channels [42]. The persistent undulation in the presence of many resonances may be a sign of Ericson fluctuations which have been studied in nuclear physics spectra [42].

³The quadratic Zeeman shift at this field sufficiently isolates the $|m_F = -21/2\rangle \leftrightarrow |m_F = -19/2\rangle$ transition from the $|m_F = -19/2\rangle \leftrightarrow |m_F = -17/2\rangle$ transition for a narrow ARP preparation.

3.3 Summary

The high density of Feshbach resonances in dysprosium requires special consideration when choosing fields for experiments and evaporative cooling. Fermionic dysprosium is especially troublesome with an ultradense spectrum above 10 G.

Of course, the dense Feshbach spectra also deserve study and theoretical consideration for their own sake. Such studies have been carried out with erbium [24, 35] and dysprosium [35, 43] with a focus on the application of random matrix theory and quantum chaos. The data presented in this chapter on the increased density of resonances at high fields in ^{161}Dy and possible Ericson fluctuations should further motivate study of these spectra.

Chapter 4

Elastic dipolar collisions

Ultracold dysprosium is the ideal atomic system for studying dipolar collisions, owing to dysprosium's large dipole moment ($10\mu_B$). The dipole-dipole interaction is typically negligible in alkali systems but can easily dominate dysprosium collisional physics. This chapter presents studies of elastic scattering with dysprosium and the effects of dipolar scattering. Inelastic dipolar collisions are then considered in Ch. 5.

The differential cross section of two bosonic dysprosium atoms with s -wave scattering length a is [44, 45]:

$$\frac{d\sigma}{d\Omega}(\mathbf{p}_{\text{rel}}, \mathbf{p}'_{\text{rel}}) = \frac{1}{2} \left| -2a - \frac{2(\hat{\mathbf{p}}_{\text{rel}} \cdot \hat{\boldsymbol{\epsilon}})^2 + 2(\hat{\mathbf{p}}'_{\text{rel}} \cdot \hat{\boldsymbol{\epsilon}})^2 - 4(\hat{\mathbf{p}}_{\text{rel}} \cdot \hat{\boldsymbol{\epsilon}})(\hat{\mathbf{p}}'_{\text{rel}} \cdot \hat{\boldsymbol{\epsilon}})(\hat{\mathbf{p}}_{\text{rel}} \cdot \hat{\mathbf{p}}'_{\text{rel}})}{1 - (\hat{\mathbf{p}}_{\text{rel}} \cdot \hat{\mathbf{p}}'_{\text{rel}})^2} a_d + \frac{4}{3}a_d \right|^2, \quad (4.1)$$

where $\hat{\mathbf{p}}_{\text{rel}}$ is the incoming relative momentum of two particles, $\hat{\mathbf{p}}'_{\text{rel}}$ is the outgoing relative momentum of two particles, $\hat{\boldsymbol{\epsilon}}$ is the dipole alignment vector, $a_d = m\mu_0\mu^2/(8\pi\hbar^2)$ is the dipole length, $\mu = 10\mu_B$ is the magnetic dipole momentum of dysprosium, and hats indicated unit vectors. We will assume all particles collide with $\hat{\mathbf{p}}_{\text{rel}} = \hat{x}$.

The structure of the differential cross section becomes apparent by plotting 2D projections of Eq. (4.1) for different field orientations (which sets the dipole alignment

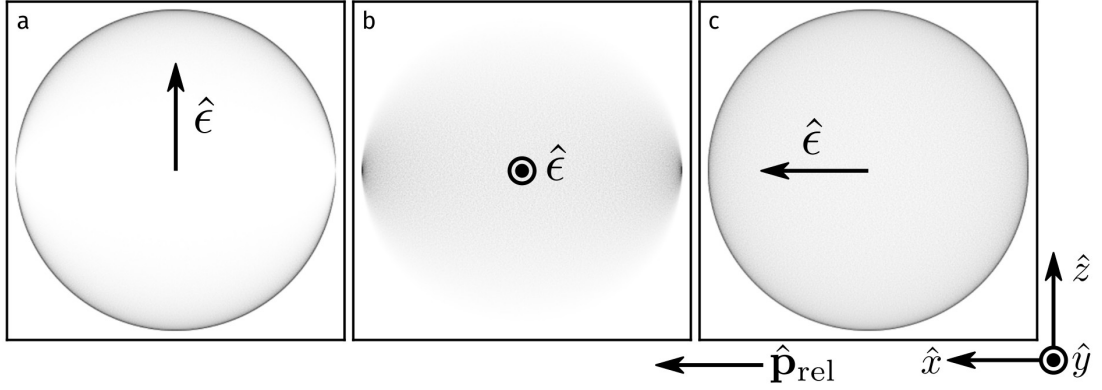


Figure 4.1: 2D projections of the differential cross section of a dipolar Bose gas from Eq. (4.1) for (a) $\hat{\epsilon} = \hat{z}$, (b) $\hat{\epsilon} = \hat{y}$, and (c) $\hat{\epsilon} = \hat{z}$.

vector) with $\hat{\mathbf{p}}_{\text{rel}} = \hat{x}$, as shown in Fig. 4.1. The first two panels are essentially rotations of each other because $\hat{\mathbf{p}}_{\text{rel}} \cdot \hat{\epsilon} = 0$ in both. In this configuration the atoms preferentially scatter into the dipole alignment direction as Eq. (4.1) simplifies to:

$$\left. \frac{d\sigma}{d\Omega}(\hat{\mathbf{p}}_{\text{rel}}, \mathbf{p}'_{\text{rel}}) \right|_{\hat{\mathbf{p}}_{\text{rel}} \perp \hat{\epsilon}} = \frac{1}{2} \left| -2a - \frac{2(\hat{\mathbf{p}}'_{\text{rel}} \cdot \hat{\epsilon})^2}{1 - (\hat{\mathbf{p}}'_{\text{rel}} \cdot \hat{\epsilon})^2} a_d + \frac{4}{3} a_d \right|^2. \quad (4.2)$$

In contrast, $\hat{\mathbf{p}}_{\text{rel}} \cdot \hat{\epsilon} = 1$ in Fig. 4.1(c), and the atoms scatter uniformly in all directions, as Eq. (4.1) simplifies to

$$\left. \frac{d\sigma}{d\Omega}(\hat{\mathbf{p}}_{\text{rel}}, \mathbf{p}'_{\text{rel}}) \right|_{\hat{\mathbf{p}}_{\text{rel}} \parallel \hat{\epsilon}} = \frac{1}{2} \left| -2a - \frac{2}{3} a_d \right|^2. \quad (4.3)$$

4.1 Imaging the differential cross section

The differential cross section of two particles can be measured by repeatedly colliding the particles and keeping track outgoing momentum after each collision. By colliding atomic clouds with many atoms, we can parallelize this process by colliding

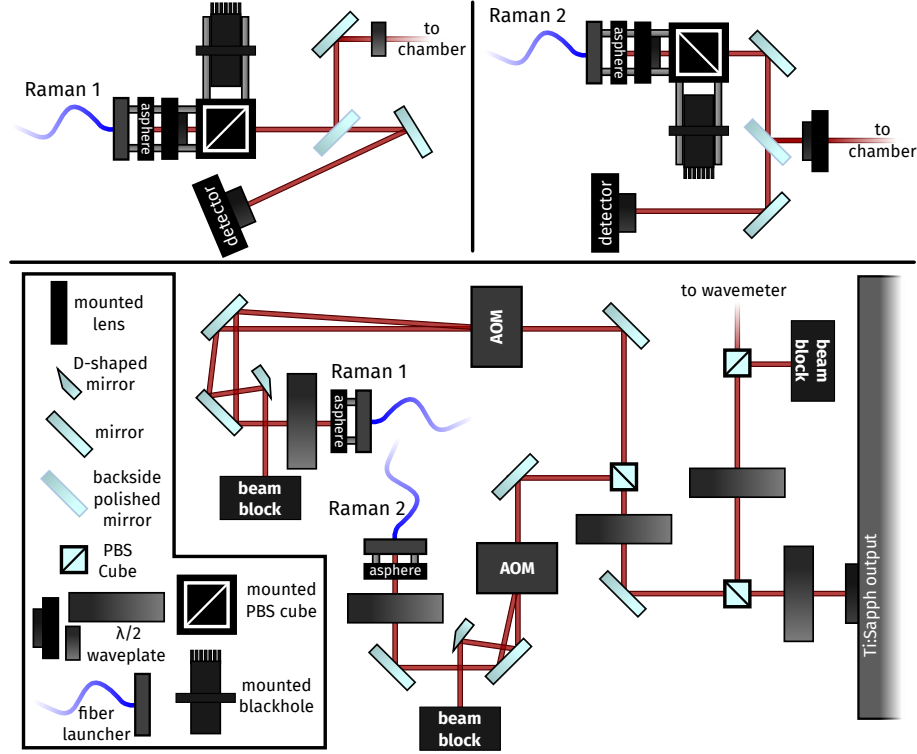


Figure 4.2: The 741-nm laser setup used for an optical lattice or Raman coupling. The $\lambda/2$ waveplates just before the chamber can be adjusted such that the beams interfere (i.e., form a lattice) or drive Raman transitions. Laser intensity is sampled via backside polished mirrors after the fibers and stabilized by feedback to the AOMs.

multiple atoms at once.

We produce a single BEC of ^{162}Dy and use an optical lattice to diffract the condensate into the $\pm 2n\hbar\mathbf{k}_L$ diffraction orders, where $\mathbf{k}_L$ is the lattice wave vector. The optical lattice is formed by two fiber-coupled 741-nm beams derived from a Ti:Sapph laser. The pre- and post-fiber optics are shown in Fig. 4.2. By using a precisely timed two-pulse sequence, the condensate can be split into the $\pm\hbar\mathbf{k}_L$ diffraction orders with high efficiency [46]. Thus, two condensates that collide with $|\mathbf{p}_{\text{rel}}| = 4\hbar\mathbf{k}_L$ are produced.

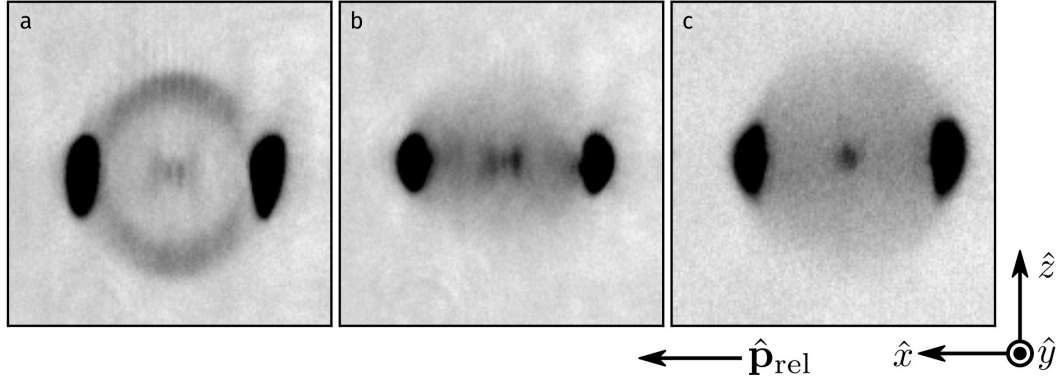


Figure 4.3: Absorption images of two ^{162}Dy BECs (the large, dark spots) after a collision with three different magnetic field (i.e., dipole alignment) directions. The halo of scattered atoms between the BECs map out the differential cross section. The orientations and projections correspond to those in Fig. 4.1 with (a) $\hat{\epsilon} = \hat{z}$, (b) $\hat{\epsilon} = \hat{y}$, and (c) $\hat{\epsilon} = \hat{x}$. Each image is the average of 40 absorption images and scaled to highlight the scattered halo.

Immediately after applying the lattice grating, all trapping potentials are removed, and the two condensates are allowed to collide and expand. Most of the atoms do not collide due to the low density of the atomic cloud, but those atoms that do form a halo of scattered atoms that reflect the differential cross section. By applying a magnetic bias field along different directions, we can realize the three different dipole alignment directions in Fig. 4.1.

The scattered clouds are presented in Fig. 4.3. The effect of the magnetic field orientation is clear and has good phenomenological agreement with the plots in Fig. 4.1. In particular, atoms do preferentially scatter along the bias field direction when the field is perpendicular to the $\hat{\mathbf{p}}_{\text{rel}}$.

However, imperfection in the applied diffraction leaves residual atoms near $0\hbar k_L$ (most clearly seen in Fig. 4.3(a)). These atoms contribute noise to the measurement both near $0\hbar k_L$ and in $|2\hbar k_L|$ halos from collisions with the $\pm 2\hbar k_L$ BECs.

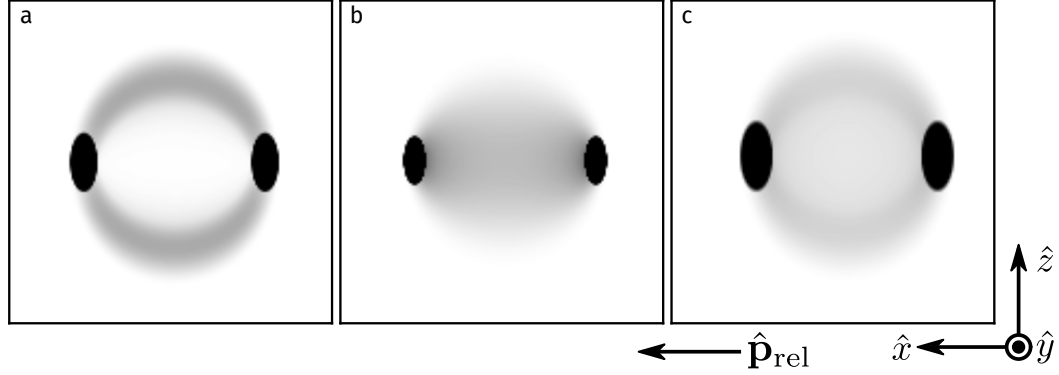


Figure 4.4: 2D projections of the differential cross section of ^{162}Dy for three different magnetic field (i.e., dipole alignment) orientations. The ideal differential cross section has been convolved with a momentum distribution to better match the images in Fig. 4.3. The orientations and projections correspond to those in Fig. 4.1 and Fig. 4.3 with (a) $\hat{\epsilon} = \hat{z}$, (b) $\hat{\epsilon} = \hat{y}$, and (c) $\hat{\epsilon} = \hat{z}$. The s -wave to dipole length ratio is set to $a/a_d = .8$, and the images are scaled to highlight the scattered cloud.

Furthermore, the momentum spread of the BECs is also present in the scattered atoms, essentially blurring the differential cross section. This can be partially included in the theory by convolving the ideally scattered clouds in Fig. 4.1 with a finite momentum spread. This assumes that the momentum distribution does not appreciably affect $\hat{\mathbf{p}}_{\text{rel}}$, which is justified by the fact that the radius of the scattered halo is much larger than the radius of the unscattered clouds.

The differential cross sections with momentum convolutions are shown in Fig. 4.4. The momentum distribution used for the convolution was fitted to both the scattered halo and the unscattered atoms simultaneously.

4.2 Analysis of the differential cross section

4.2.1 Direct fitting of scattering images

One use of this scattering data would be to fit the s -wave scattering length a in Eq. (4.1). By measuring the amount of isotropic scattering due to a relative to the anisotropic scattering due to a_d , we can extract an absolute value for a .

The data with $\hat{\epsilon} = \hat{y}$ is best suited for this purpose because the most sensitive region to a/a_d —the outer edge of the scattered halo where a_d hardly scatters—is relatively noise free due to the larger density of states (after the 2D projection) and the lack of the residual atoms. By contrast, what would be the most sensitive region in the $\hat{\epsilon} = \hat{z}$ data—the center of the halo—is actually the noisiest. The $\hat{\epsilon} = \hat{x} = \hat{\mathbf{p}}_{\text{rel}}$ data is of no use because the a_d scatters isotropically and is indistinguishable from a .

We tested the fitting routine against simulation data provided by Andrew Sykes using the direct simulation Monte Carlo (DSMC) method for dipolar scattering [45]. The simulations were run with different field configurations and ratios of a/a_d . The fits perform best on the $\hat{\epsilon} = \hat{y}$ simulations as expected, and when the ratio a/a_d is larger than ~ 0.6 the fits capture a/a_d to within 5%.

By fitting the $\hat{\epsilon} = \hat{y}$ data, we obtain a value of $a/a_d = .79$ or $a = 153(8)a_0$ with the error based on the simulation fits. This is consistent with recent measurements of the ^{162}Dy scattering length via anisotropic expansion of a thermal cloud.

The free parameters in fitting the images are the collision center (x_0 and z_0), the Thomas-Fermi radii of the BECs (R_x and R_z), the s -wave-to-dipole length ratio (a/a_d), the total number of scattered atoms (N_s), and the total number of unscattered atoms (N_u). The $\hat{\epsilon} = \hat{z}$ data is highly sensitive to initial fitting parameters and does

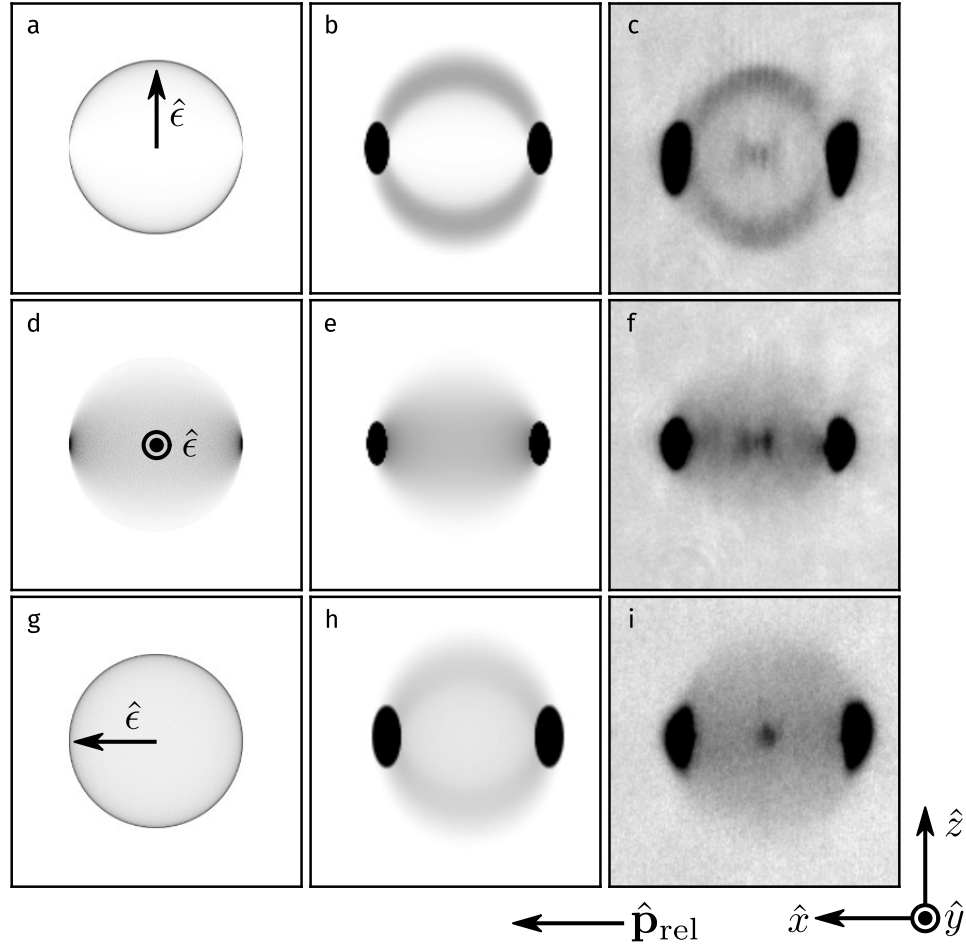


Figure 4.5: A composite presentation of the images in Fig. 4.1 (left column), 4.4 (middle column), and 4.3 (right column). The magnetic field (i.e., dipole alignment) orientation is (a)-(c) $\hat{\epsilon} = \hat{z}$, (d)-(f) $\hat{\epsilon} = \hat{y}$, and (g)-(i) $\hat{\epsilon} = \hat{x}$. The theory images are plotted with $a/a_d = .8$.

not converge stably. According to the theory, the $\hat{\epsilon} = \hat{x}$ data does not depend on a/a_d , and no fit was attempted.

Figure 4.5 is a composite presentation of all the theory and experimental images.

4.2.2 Fitting the ratio of scattered atoms

The approach above relied on the distribution of the atoms in the scattered. Alternatively, we can consider the fraction of atoms in the scattered halo as a function of the angle between $\hat{\mathbf{p}}_{\text{rel}}$ and the dipole alignment, $\eta = \cos^{-1}(\hat{\mathbf{e}} \cdot \hat{\mathbf{p}}_{\text{rel}})$. This depends on the total cross section, which is derived in Ref. [44]:

$$\sigma_B(a) = \frac{\pi}{9} \left\{ 72a^2 - 24aa_d [1 - 3\cos^2(\eta)] + a_d^2 [11 - 30\cos^2(\eta) + 27\cos^4(\eta)] \right\}. \quad (4.4)$$

The probability of a single atom scattering with the counter-moving cloud is on the order of $n_0 R_x \sigma_B$, where n_0 is the peak density of the BEC and R_x is the Thomas-Fermi radius of the BEC; thus, the expected number of scattering events is on the order of $N_S = n_0 R_x \sigma_B \times N$.

However, we can only detect the number of scattered atoms N_H and cannot detect the number of scattering events: A single atom could undergo multiple collisions, causing undercounting of scattering events. This undercounting is analogous to taking a sample of size N_S from a sample of N with replacement and discarding the repeated selections¹. With this understanding, the expected number of repeated selections N_R can be shown to be

$$N_R = N_S + N \left(\frac{N-1}{N} \right)^{N_S} - N \quad (4.5)$$

and the expected number of atoms in the scattered halo is $N_H = N_S - N_R$.

We have data for seven different field configurations that cover five different values of η . The average atom number varies slightly across the datasets so we first renormalize each dataset to the average number across all datasets. The image regions

¹“Repeated selections” are typically referred to as “collisions,” but we will avoid that language here for obvious reasons.

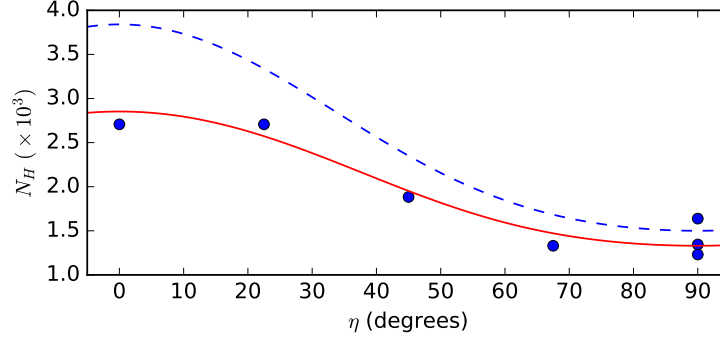


Figure 4.6: Number of total scattered halo atoms N_H for different angles between the $\hat{\mathbf{p}}_{\text{rel}}$ and dipole alignment $\hat{\mathbf{e}}$, $\eta = \cos^{-1}(\hat{\mathbf{e}} \cdot \hat{\mathbf{p}}_{\text{rel}})$. The red curve is a fit to the data and gives $a = 140a_0$. The blue curve is a plot of the same parameters but with no sampling-with-replacement correction.

with unscattered atoms are removed and the remaining atoms summed. The totaled atoms are fit to the function

$$f(a, \gamma) = N_s(a, \gamma) - N_R(a, \gamma) = N \left[1 - \left(\frac{N-1}{N} \right)^{\sigma_B(a)N^2\gamma} \right] \quad (4.6)$$

with free parameters a and γ , where $N\gamma$ is the collisional density. The fitted parameters are $a = 140a_0$ and $\gamma = 3.4 \times 10^9 \text{ m}^{-2}$. As expected, $N\gamma$ is on the order of $n_0 R_x$. The data and fit (in red) are shown in Fig. 4.6. The blue curve shows the plot with the same parameters but with no correction from N_R .

4.3 Cross-dimensional relaxation

Another method of probing elastic collisions is by observing the rate at which collisions redistribute energy between otherwise orthogonal degrees of freedom within

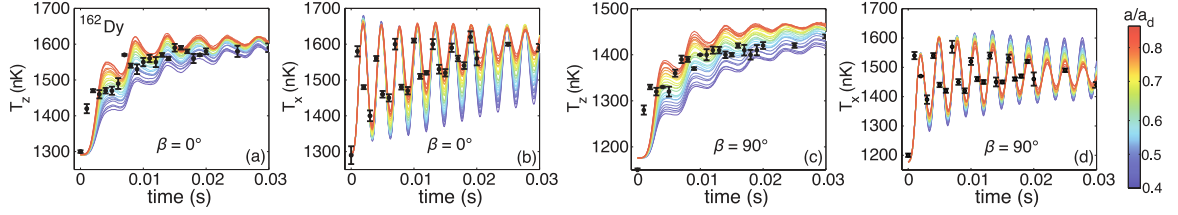


Figure 4.7: A comparison between the experimentally measured and simulated rethermalization of ^{162}Dy . Panels (a) and (b) show the rethermalization dynamics for $\beta = 0^\circ$; panels (c) and (d) for $\beta = 90^\circ$. In each plot the data points with error bars correspond to experimental measurements. The multiple solid lines (each with a different color) correspond to simulation results, with the simulated scattering length indicated by color.

a gas. In a non-interacting, harmonically confined gas, energy in one dimension cannot be transferred to the other two. If the gas begins with an energy imbalance between two dimensions, that imbalance will persist indefinitely. However, interactions mediate collisions which provide a channel for energy exchange, and the two dimensions will eventually equilibrate.

The strength of the interactions present determines the frequency of collisions which in turn sets the rate of equilibration. Thus, the collision rate can be measured by preparing a gas such that on average the atoms have more energy along one single dimension and observing the leakage of energy from that dimension to the others. This technique is called cross-dimensional relaxation [47] and is a common method for measuring collision rates in ultracold gases.

The analysis of cross-dimensional relaxation is especially simple for bosonic alkali atoms because the scattering cross section is essentially due to isotropic interactions and dominated by the s -wave scattering length [48]. However, for dysprosium the s -wave interaction and dipole-dipole interaction are of comparable strength, and the cross section is anisotropic as shown in Eq. (4.4) [44]. By controlling observing the

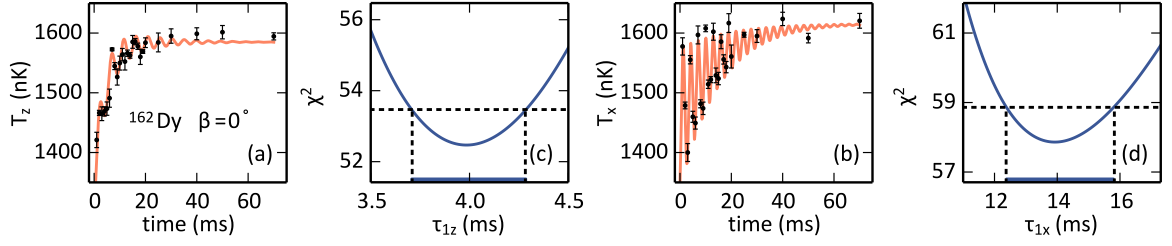


Figure 4.8: Fits to ^{162}Dy data with $\beta = 0^\circ$. The data points in (a) and (b) are the experimental measurements, and the solid line shows the best fit using Eq. (4.7). Panels (c) and (d) demonstrate the χ^2 analysis to determine fit parameter uncertainties. A parameter [e.g., τ_{1z} in (c)] is manually varied, and the 1σ uncertainty (shown by the blue bar) corresponds to the optimized χ^2 increasing by 1 [49, 50].

rethermalization with different dipole orientations, the s -wave scattering and dipolar scattering contributions can be distinguished. Furthermore, by comparing the experiments with rethermalization simulations—again performed by Andrew Sykes using the DSMC method—a better measurement can be made.

Cross-dimensional relaxation was performed with thermal ^{162}Dy and ^{164}Dy in crossed ODTs after evaporation. The trap frequencies were initially $[\omega_x, \omega_y, \omega_z] = 2\pi \times [151(2), 70(5), 393(1)]$ Hz. The dipole orientation was set by a bias field at angle β from $\hat{z}$. In order to prepare a non-equilibrium energy distribution, energy was added along one dimension of the trap by diabatically compressing the trap to $[\omega_x, \omega_y, \omega_z] = 2\pi \times [175(3), 103(5), 393(1)]$ Hz. This adds energy primarily along y , though some energy is also added along x . The observed and simulated evolutions of the temperature² along x and z are shown in Fig. 4.7 for two different field orientations. Uncertainty in these data are given as 1σ standard errors and are dominated by statistical fluctuations.

²Although the cloud is not in equilibrium, the term “temperature” is used to describe the kinetic energy in each dimension for convenience.

The simulations qualitatively capture the experimental data. The diabatic compress excites collective modes in the cloud due to the cloud being in the hydrodynamic regime. This collective mode excitation appears as oscillations in T_x and T_z in both the experiment and simulation. The phase offset between the data and simulation is likely due to experimental uncertainty in the trap parameters.

The hydrodynamic regime is especially hard to avoid with dysprosium due to dysprosium's large scattering cross section that is a combination of s -wave scattering and dipolar scattering. Eliminating the oscillations would require lowering the cloud density by weakening the trap frequencies; however, this would also lower the trap depth and make the experiments susceptible to plain evaporation.

In order to make a quantitative comparison between experiment and simulation, the data along each axis was fit to the function

$$\tilde{T}(t) = \tilde{T}_f + (\tilde{T}_i - \tilde{T}_f)e^{-t/\tau_1} + \tilde{A}e^{-t/\tau_2} \sin[2\tilde{\omega}t - \tilde{\delta}] \quad (4.7)$$

with free parameters $\tilde{T}_i$ and $\tilde{T}_f$ related to the initial and final temperatures, respectively; τ_1 as the rethermalization time constant; and $\tilde{A}$, τ_2 , $\tilde{\omega}$, and $\tilde{\delta}$ as related to the damped oscillations. This function captures the experiments and simulations well with a reduced- χ^2 of order unity for both.

The error function $\sum_k [\tilde{T}(t_k) - T(t_k)]^2$ is minimized for x and z independently near the physically meaningful values of the free parameters. Thus, we extract rethermalization and damping timescales τ_{1x} and τ_{2x} along x and timescales τ_{1z} and τ_{2z} along z . The fit uncertainties are obtained through the standard χ^2 analysis: The parameter in question is fixed and manually adjusted around the optimal value (with all other parameters free) until the χ^2 —not the reduced- χ^2 —increases by 1 [49, 50]. Example

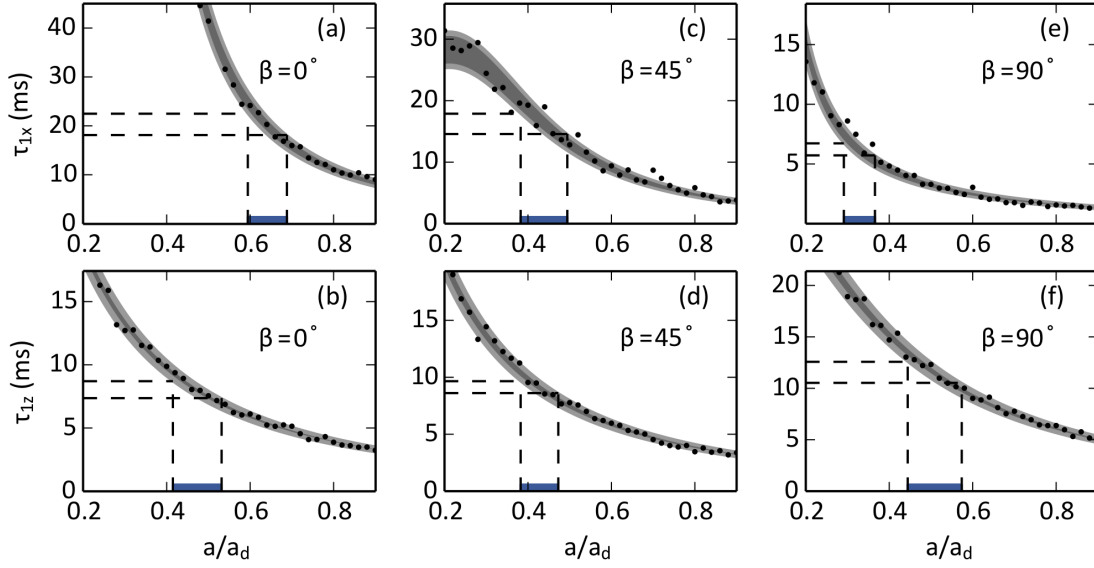


Figure 4.9: Analyses of ^{164}Dy data. The dots show the value of τ_1 extracted by fitting the functional form Eq. (4.7) to the simulation results. The dark grey band denotes a 1σ uncertainty from the simulation, and the larger grey band includes experimental uncertainty. The horizontal dashed lines show the bounds at 1σ uncertainty of τ_1 from experimental data. The blue bar along the bottom axis of each figure shows the 1σ estimation of a/a_d , i.e., where the grey area lies between the 1σ experimental bounds. Panels (a) and (b) correspond to $\beta = 0^\circ$, (c) and (d) show $\beta = 45^\circ$, and (e) and (f) show $\beta = 90^\circ$.

fits and χ^2 analyses are shown in Fig. 4.8. Only the rethermalization times are tightly constrained by the data.

The bounded rethermalization timescales for ^{164}Dy are presented in Fig. 4.9 and clearly change with varying dipole alignment angle β , reflecting the change in dipolar cross section. These timescales were connected to the ratio a/a_d through Andrew Sykes's simulations. The dark gray bands in Fig. 4.9 represent the simulation errors, and the light gray bands represent the combined errors from the simulation and trap frequency uncertainty.

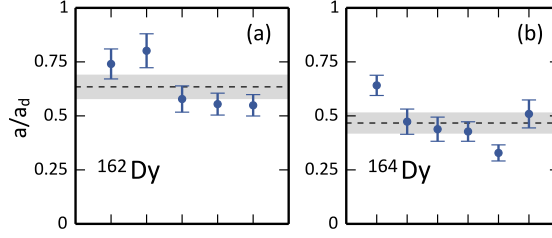


Figure 4.10: Summaries of the measured scattering lengths extracted from each individual experiment along with the weighted average (dashed line) and its 1σ error (grey band). Results for ^{162}Dy and ^{164}Dy are shown in (a) and (b), respectively. The weighted averages and 1σ standard errors are $a/a_d = 0.63(5)$ for ^{162}Dy and $a/a_d = 0.47(4)$ for ^{164}Dy .

Each rethermalization timescale is treated as an independent of a/a_d from which the value of a can be extracted. The results for ^{162}Dy and ^{164}Dy are shown in Fig. 4.10. The weighted average for ^{162}Dy is $122(10)a_0$ and for ^{164}Dy is $92(8)a_0$.

4.4 Summary

Elastic dipolar collisions have been studied with dysprosium via direct measurement of the dipolar differential cross section and via rethermalization experiments. Both techniques reveal the anisotropy of the dipole-dipole interaction. However, one aspect not highlighted in these experiments is the long-range, $1/r^3$ nature of the interaction. Future experiments in lower dimensions similar to experiments in Ref. [51] will probe the effect of long-range elastic collisions on thermalization in lower dimensions.

Chapter 5

Inelastic dipolar collisions

Only if the magnetic momentum of an atom is maximally aligned with the (local) external field can the atom be considered in the absolute ground state. This alignment—and the alignment of the atom’s spin along any axis for that matter—is quantized and given by the spin projection operator. For an atom with spin F , the allowable $2F + 1$ unique projections define the magnetic Zeeman levels of the atom, and the energy difference between these Zeeman states gives rise to the Zeeman effect.

In addition to elastic dipolar collisions, the dipole-dipole interaction also supports inelastic collisions, which change an atom’s Zeeman state. If the atom transitions into a higher energy Zeeman state, kinetic energy is absorbed into internal energy of the atom. More often, the atom transitions to a lower Zeeman energy and the lost internal energy is instead released as kinetic. Such exothermic dipolar collisions are called dipolar relaxation and are a source of heating or atom loss in ultracold atomic gases.

Dipolar relaxation among highly magnetic atoms in magnetostatic traps were considered in the context of bosonic Cr gases at fifty to hundreds of μK [39] and at

a few hundred nK [52] and Dy gases at hundreds of mK [53] and at a few hundred μK [30]. The authors of Ref. [39] derived a universal expression for inelastic dipolar scattering using the first-order Born approximation and observed rapid collisional loss of bosonic Cr consistent with this theory¹. That Fermi statistics might suppress dipolar relaxation was first discussed in Ref. [52].

By measuring dipolar relaxation rates, we find that spin relaxation is enhanced among bosons while suppressed among fermions, supporting the conclusion that quantum statistics play a substantial role in these collisions. The strikingly close correspondence between our spin relaxation data and theory—with no free parameters—is a clear manifestation of universal inelastic dipolar scattering.

5.1 First-order Born approximation

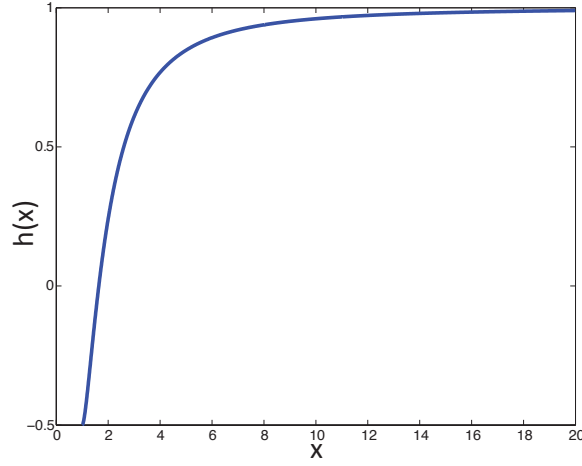
Following Refs. [39, 52], we now describe two-particle dipolar scattering within the first-order Born approximation. Given an initial two-body spin state that is maximally stretched and weak-field-seeking $|F, m_F = +F\rangle|F, m_F = +F\rangle$, the cross sections for the elastic (σ_0), single-spin-flip inelastic (σ_1), and double-spin-flip inelastic (σ_2) processes are [39, 52]:

$$\sigma_0 = \frac{8\pi}{15} \left(\frac{2}{3}\right) F^4 \left(\frac{\mu_0(g_F\mu_B)^2 m}{4\pi\hbar^2}\right)^2 [1 + \epsilon h(1)], \quad (5.1)$$

$$\sigma_1 = \frac{8\pi}{15} F^3 \left(\frac{\mu_0(g_F\mu_B)^2 m}{4\pi\hbar^2}\right)^2 [1 + \epsilon h(k_{f1}/k_i)] \frac{k_{f1}}{k_i}, \quad (5.2)$$

$$\sigma_2 = \frac{8\pi}{15} F^2 \left(\frac{\mu_0(g_F\mu_B)^2 m}{4\pi\hbar^2}\right)^2 [1 + \epsilon h(k_{f2}/k_i)] \frac{k_{f2}}{k_i}, \quad (5.3)$$

¹See Ref. [48] and references therein for prior work on inelastic dipolar scattering.

Figure 5.1: The function $h(x)$

where $k_i = \mu v_{rel}/\hbar$ is the initial relative momentum of the two atoms, $k_{f\eta}$ is the final relative momentum, $\mu = m/2$ is the reduced mass, and g_F is the g -factor². The final relative momentum $k_{f\eta}$ must satisfy the conservation of energy condition $(\hbar k_{f\eta})^2 = (\hbar k_i)^2 + \eta \times (2\mu\Delta E)$, where $\Delta E = g_F\mu_B B$ is the Zeeman energy in a magnetic field B . Note that $\sigma_1/\sigma_2 \approx F^3$ and that for large-spin atoms such as ^{162}Dy ($F = 8$) and ^{161}Dy ($F = 21/2$) single-spin-flip processes typically dominate.

The kinematic factors in Eq. (5.2) and (5.3) are a function of the ratio of output to input relative momenta $k_{f\eta}/k_i$. We do not consider endothermic collisions, and therefore $k_{f\eta}/k_i$ is limited to the interval $[1, \infty)$. At large B and low T , $k_{f\eta}/k_i \propto \sqrt{B/T}$.

The term $h(k_{f\eta}/k_i)$ is the ratio of the exchange to the direct terms in the cross section:

$$h(x) = -\frac{1}{2} - \frac{3}{8x} \frac{(1-x^2)^2}{1+x^2} \ln \left(\frac{(1-x)^2}{(1+x)^2} \right). \quad (5.4)$$

²The g -factor is 1.242 for ^{162}Dy and 0.946 for ^{161}Dy [54].

³This relation is only approximately true because the ratio $k_{f1} \neq k_{f2}$.

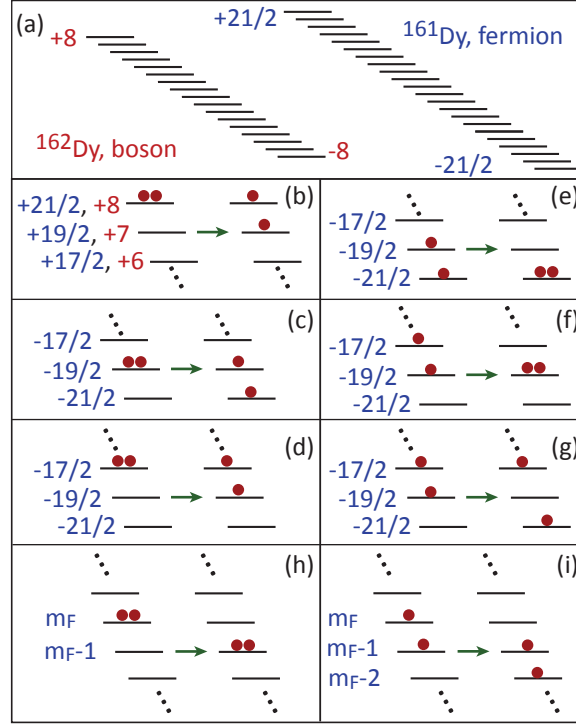


Figure 5.2: (a) Zeeman m_F sublevels of the relevant Dy ground states. Numbers indicate the maximally stretched m_F states. (b–d) Single-spin-flip dipolar relaxation of spin-polarized states into spin mixtures. Arrow points from the incoming to the outgoing spin population. (e) and (f) Single-spin-flip dipolar relaxation of spin mixtures into spin-polarized states. (g) Single-spin-flip dipolar relaxation of a spin mixture into a spin mixture. (h) Double-spin-flip dipolar relaxation of spin-polarized states to spin-polarized states. (i) Double-spin-flip dipolar relaxation of spin-mixture to different spin-mixture.

This ratio monotonically increases from $h(1) = -1/2$ to $h(x \rightarrow \infty) = 1 - 4/x^2$ as shown in Fig. 5.1 [39].

The quantum statistics of the colliding particles are captured in the value of ϵ : +1 for indistinguishable bosons, -1 for indistinguishable fermions, and 0 for distinguishable particles. As long as the colliding particles have identical spin states in either the incoming or outgoing channel [48], indistinguishable statistics ($\epsilon = \pm 1$) apply. Thus,

the processes depicted in Fig. 5.2(b)-(f) and (h) have $\epsilon = \pm 1$, and the processes in Fig. 5.2(g) and (i) have $\epsilon = 0$.

The different values of ϵ yield different scalings of the inelastic cross section in the large B , low T limit. The cross section vanishes as $4\sqrt{T/B}$ for $\epsilon = -1$, increases as $2\sqrt{B/T}$ for $\epsilon = +1$, and increases as $\sqrt{B/T}$ for $\epsilon = 0$. The relative fermionic suppression ratio in this limit is therefore $\sigma_1^{\text{fermions}}/\sigma_1^{\text{bosons}} = 2T/B$.

5.1.1 General cross sections for $|m_F| \leq F$

The cross sections in Eq. (5.1), (5.2), and (5.3) only apply to stretched-state collisions with $m_F = F$. The general case is written most conveniently by first removing the contribution from spin operators:

$$\sigma(k_{f\eta}/k_i, \epsilon) = \frac{8\pi}{15} \left(\frac{\mu_0(g_F\mu_B)^2 m}{4\pi\hbar^2} \right)^2 [1 + \epsilon h(k_{f\eta}/k_i)] \frac{k_{f\eta}}{k_i}. \quad (5.5)$$

where $k_{f\eta}$ satisfies the conservation of energy condition $(\hbar k_{f\eta})^2 = (\hbar k_i)^2 + \eta(2\mu\Delta E)$. Note that this form even holds for the elastic case of Eq. (5.1) with $k_{f0}/k_i = 1$.

The elastic, one-flip, and two-flip processes differ only by the spin operators connecting the incoming and outgoing spinors:

$$\sigma_0 = \left| \sqrt{\frac{2}{3}} \langle \text{out} | F_{1z} F_{2z} | \text{in} \rangle \right|^2 \sigma(k_{f0}/k_i, \epsilon) \quad (5.6)$$

$$\sigma_1 = \left| \frac{1}{2} \langle \text{out} | (F_{1z} F_{2-} + F_{1-} F_{2z}) | \text{in} \rangle \right|^2 \sigma(k_{f1}/k_i, \epsilon) \quad (5.7)$$

$$\sigma_2 = \left| \frac{1}{2} \langle \text{out} | F_{1-} F_{2-} | \text{in} \rangle \right|^2 \sigma(k_{f2}/k_i, \epsilon) \quad (5.8)$$

The states $|\text{in}\rangle$ and $|\text{out}\rangle$ must be properly symmetrized if the collision involves

identical particles. Appendix A contains examples of calculating cross sections from Eq. (5.7) and (5.8) for various cases.

5.1.2 Indistinguishable and distinguishable statistics

As stated above, indistinguishable statistics apply if the incoming *or outgoing* particles are identical. This may be somewhat counter-intuitive in the case of incoming distinguishable fermions. Such particles could initially collide on an *s*-wave, seemingly obviating the *p*-wave barrier and fermionic suppression.

However, dipolar collisions occur under the selection rule $\Delta l = 0, \pm 2$, where l is the orbital angular momentum. This selection rule means relaxation events cannot involve incoming and outgoing partial waves of different parity. Indistinguishable fermions in an outgoing channel must occupy an odd partial wave; therefore, even distinguishable fermions must occupy an odd partial wave if they relax to indistinguishable fermions.

5.2 Measurement of dipolar relaxation

5.2.1 Identical particles

To measure the collisional loss rates, ultracold gases of ^{162}Dy and ^{161}Dy are prepared by laser cooling in two magneto-optical-trap stages and by forced evaporative cooling in a 1064-nm crossed optical dipole trap. The temperatures of the boson and fermion gases, ~ 400 nK, were chosen to be slightly above quantum degeneracy to eliminate correlation effects [52]: $T/T_c = 1.5(1)$ [density $3(1) \times 10^{13} \text{ cm}^{-3}$] and $T/T_F = 1.4(1)$ [$7(2) \times 10^{12} \text{ cm}^{-3}$]⁴. Adiabatic rapid passage (ARP) while in the

⁴All errors represent one standard error.

optical dipole trap polarizes the atomic cloud in its absolute internal ground state. Co-trapping ^{162}Dy with ^{161}Dy was used to enhance fermionic evaporation efficiency, after which the bosons were removed from the trap by a 1-ms pulse with a 421-nm resonant pushing beam with no adverse effect on the fermions.

The atoms were then prepared in the desired Zeeman substate(s) by driving rf transitions. For ^{162}Dy , this is limited to only the stretched states $|F = 8, m_F = \pm 8\rangle$ due to the lack of nuclear spin and therefore quadratic Zeeman shifts in bosonic Dy.

The nuclear spin $I = 5/2$ of ^{161}Dy gives rise to a significant quadratic Zeeman shift at moderate magnetic fields [29]. To populate the $|F, m_f\rangle = |21/2, -19/2\rangle$ or $|F, m_f\rangle = |21/2, -17/2\rangle$ state with rf-pulse sequences, the magnetic field is switched to 18 G, providing a 20-kHz differential energy shift due to the quadratic contribution of the Zeeman shift. A narrow ARP sequence (i.e., one with a frequency sweep width less than 20 kHz) allows us to transfer the full atomic population from the ground state into the $|F, m_f\rangle = |21/2, -19/2\rangle$ state. A second narrow ARP pulse allows us to transfer the atoms into the $|F, m_f\rangle = |21/2, -17/2\rangle$ state. After state preparation, the magnetic field is switched to the desired low-field value. We observed an atom number reduction of a factor of two by the end of these preparation sequences, perhaps due to crossing a large number of the Dy Feshbach resonances while sweeping the magnetic field [23].

Stern-Gerlach measurements were used to verify the final state purity. A large and switchable magnetic field gradient is applied to vertically separate the m_F states of dysprosium so that we may image each population in a Stern-Gerlach type of experiment. The vertical ODT beam is kept on during the ballistic expansion to guide and increase the atomic densities during the separation. This increases the

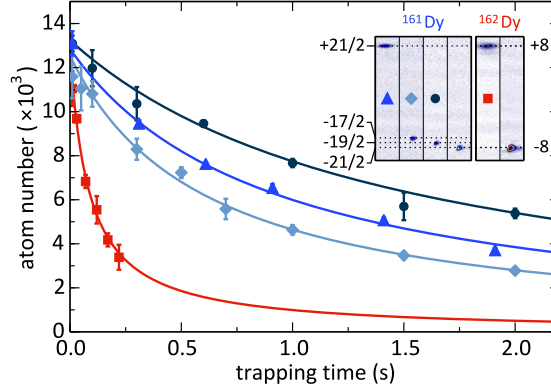


Figure 5.3: Population decay of fermionic ^{161}Dy at $B = 0.410(5)$ G, $T = 390(30)$ nK, and $\bar{n}_0 = 7(2) \times 10^{12} \text{ cm}^{-3}$ for $m_F = +21/2$ (triangles), $-19/2$ (circles), and $-17/2$ (diamonds) as well as bosonic ^{162}Dy at $B = 0.100(5)$ G, $T = 450(30)$ nK, and $\bar{n}_0 = 3(1) \times 10^{13} \text{ cm}^{-3}$ for $m_F = +8$ (squares). The solid curves are fits to the data using Eq. (5.9). (Inset) Stern-Gerlach images of initial states.

signal-to-noise ratio of the measured population and allows for longer time-of-flights that provide larger m_F -state separation.

After state preparation, the atomic cloud is trapped for varying lengths of time in order to measure population decay. Decay curves are fit to a numerically integrated rate equation that includes collision terms for both one-body loss due to background gas γ and two-body loss β_{dr} :

$$\frac{dN}{dt} = -\gamma N - \beta_{dr} \bar{V}^{-1} N^2, \quad (5.9)$$

where $\bar{V} = \sqrt{8}(2\pi)^{3/2} \sigma_x \sigma_y \sigma_z$ is the mean collisional volume for a harmonically trapped thermal cloud with Gaussian widths σ_i . The decay rate is characterized by the lifetime $\tau_{dr} = (\beta_{dr} \bar{n}_0)^{-1}$, where $\bar{n}_0 = N_0/\bar{V}$ is the initial mean collisional density.

	$\beta_{dr}^{-17/2;-17/2}$	$\beta_{dr}^{-19/2;-19/2}$	$\beta_{dr}^{-17/2;-19/2}$	$\beta_{dr}^{-19/2;-21/2}$
Expt.	10(2)	4.1(7)	60(30)	3(1)
Theor.	6.3(3)	4.1(1)	37(1)	4.2(5)

Table 5.1: Collisional loss rates in units of $[\times 10^{-13} \text{ cm}^3 \text{ s}^{-1}]$.

Note that the inelastic cross sections of Eq. (5.2) and (5.3) are not directly measured in experiments. Instead, the measured collisional loss rate β_{dr} is related to the total relaxation cross section

$$\beta_{dr} = \left\langle \left[\sum_i \frac{g_n^i \sigma_1^i(k_{f1}/k_i, \epsilon)}{g_d^i} + \sum_j \frac{g_n^j \sigma_2^j(k_{f2}/k_i, \epsilon)}{g_d^j} \right] v_{\text{rel}} \right\rangle_{\text{thermal}}, \quad (5.10)$$

where the sums are over all the possible processes contributing to the collisional loss rate and the thermal average is over the relative velocity v_{rel} . The factors g_d in the denominators account for double counting in two-body-loss terms representing collisions between identical, spin-polarized incoming particles [52, 55]: For identical particles $g_d = 2$, and for distinguishable particles $g_d = 1$. The factors g_n in the numerators account for processes in which either one⁵ ($g_n = 1$) or two ($g_n = 2$) atoms are lost after an inelastic collision [38, 39, 52].

Typical decay curves for four different spin-polarized ensembles are shown in Fig. 5.3. The fermions are prepared in either the $m_F = +21/2$, $-19/2$, or $-17/2$ state, as in Fig. 5.2(b–d), respectively, and the bosons are prepared in the $m_F = +8$ state as in Fig. 5.2(b). The inset of Fig. 5.3 contains Stern-Gerlach-separated images of these states as well as the absolute ground states $m_F = -21/2$ and $m_F = -8$. Decay of these states, which cannot undergo dipolar relaxation, are not presented

⁵The single atom loss case applies to the individual spin populations during a collision involving an initial spin mixture.

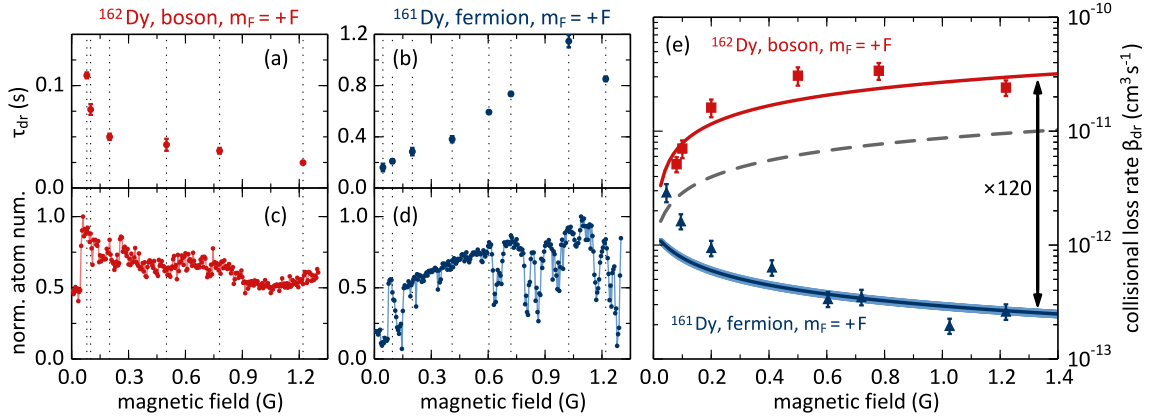


Figure 5.4: Dipolar relaxation as depicted in Fig. 5.2(b) for bosonic ^{162}Dy ($m_F = +8$) and fermionic ^{161}Dy ($m_F = +21/2$). (a)-(b) Two-body-loss lifetimes versus magnetic field. (c)-(d) Feshbach spectra presented as normalized atom loss. (e) Two-body collisional loss rates for ^{162}Dy (squares) and ^{161}Dy (triangles) at the same fields as in (a)-(d). Curves are collisional loss rates calculated using the expressions for σ_1 and σ_2 in Eq. (5.2) and (5.3), respectively, and correspond to ^{162}Dy (top), ^{161}Dy (bottom) and distinguishable particles (middle) at $T = 450(30)$ nK (top, middle) and $390(30)$ nK (bottom) with no free parameters. Thickness represents temperature error.

due to their much longer lifetimes, limited only by $1/\gamma = 21(1)$ s. Table 5.1 lists the experimental decay rates β_{dr} for the $m_F = -19/2, -17/2$ cases, along with the corresponding theory predictions for each initial state measured.

As described earlier, the bosonic lifetime is expected to decrease as the magnetic field increases, while the fermionic lifetime should increase. Both trends are observed, as shown in Fig. 5.4(a) and (b). While bosonic ^{162}Dy decays rapidly, the fermionic gases at 1 G live for ~ 1 s at this density. Figure 5.4(e) presents the β_{dr} 's of the data in Figs. 5.4(a) and (b). Data are in remarkable agreement with the theory curves, though the discrepancy of the fermion β_{dr} 's at fields below ~ 0.2 G warrants further investigation.

The high density of Feshbach resonances in Dy could mask the universal nature

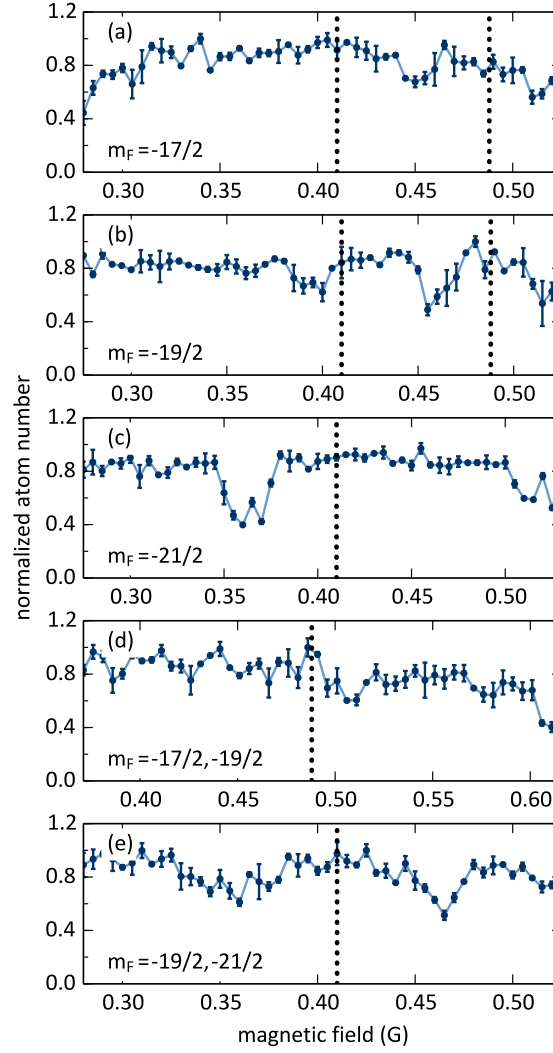


Figure 5.5: Feshbach spectra for ^{161}Dy (a)-(c) spin states and (d)-(e) spin mixtures. All spectra were taken at temperatures between 400-500 nK. Dotted lines indicate fields at which dipolar relaxation data was taken.

of Eq. (5.2) by introducing additional, three-body loss channels. Feshbach spectra for the different m_F states were measured prior to investigating the magnetic field dependence of dipolar relaxation, and magnetic fields were selected to minimize the effect of Feshbach resonances in the data of Figs. 5.3–5.6. Feshbach spectra for the

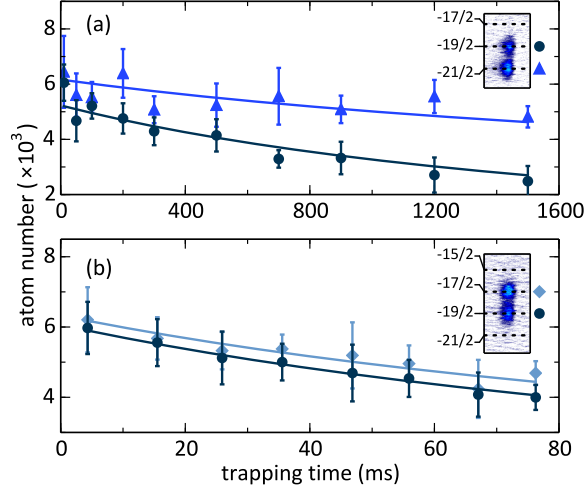


Figure 5.6: Curves are fits to coupled two-body-loss rate equations. (a) Population decay of fermionic ^{161}Dy in the $m_F = -19/2$ (circles) and $m_F = -21/2$ (triangles) states at $T = 450(20)$ nK and $B = 0.410(5)$ G. (b) Population decay of ^{161}Dy in the $m_F = -17/2$ (diamonds) and $m_F = -19/2$ (circles) states at $T = 380(20)$ nK and $0.488(5)$ G. Initial density of each spin state is $2(1) \times 10^{12} \text{ cm}^{-3}$. (Insets) Averages of 18 Stern-Gerlach images.

$m_F = +8$ bosons and $m_F = +21/2$ fermions are shown in Fig. 5.4 (c) and (d), respectively, and additional spectra are shown in Fig. 5.5.

5.2.2 Distinguishable particles

The fermions are also prepared in spin mixtures to measure interspecies dipolar relaxation rates. State preparation is the same as described above, but after preparing the atoms in $|21/2, -19/2\rangle$, a resonant rf $\pi/2$ -pulse of duration $250 \mu\text{s}$. By applying the correct frequency, the atoms can be prepared in the mixture $|21/2, -17/2\rangle|21/2, -19/2\rangle$ or $|21/2, -19/2\rangle|21/2, -21/2\rangle$.

The spin populations are co-trapped and subsequently separated and imaged via a Stern-Gerlach measurement. The populations are fit to the coupled rate equations

describing the population evolution of atoms in spin states a and b coupled by an interspecies scattering term:

$$\begin{aligned}\frac{dN_a}{dt} &= -\gamma N_a - \beta_{dr}^a \frac{N_a^2}{\bar{V}} - \beta_{dr}^{ab} \frac{N_a N_b}{\bar{V}} \\ \frac{dN_b}{dt} &= -\gamma N_b - \beta_{dr}^b \frac{N_b^2}{\bar{V}} - \beta_{dr}^{ba} \frac{N_b N_a}{\bar{V}}.\end{aligned}\tag{5.11}$$

The intraspecies loss rates are fixed by the single-species measurements. The free parameters for the fitting procedure are the initial atom numbers N_a^0 and N_b^0 and the interspecies loss rate $\beta_{dr}^{ab} = \beta_{dr}^{ba}$.

As predicted by theory, we observed suppression in the decay of the $m_F = -\frac{19}{2}, -\frac{21}{2}$ mixture but no suppression in the decay of the $m_F = -\frac{17}{2}, -\frac{19}{2}$. The measured and theoretical β_{dr} 's are listed in Table 5.1.

The drastically different decay rates are due to the different statistics governing the dominant processes. The only interspecies decay channel available to the $m_F = -\frac{19}{2}, -\frac{21}{2}$ mixture is $|-19/2\rangle|-21/2\rangle \rightarrow |-21/2\rangle|-21/2\rangle$, as depicted in Fig. 5.2(d). This process results in indistinguishable particles and therefore exhibits fermionic suppression ($\epsilon = -1$). In contrast, the decay of the $m_F = -\frac{17}{2}, -\frac{19}{2}$ mixture is dominated by the process $|-17/2\rangle|-19/2\rangle \rightarrow |-17/2\rangle|-21/2\rangle$ involving distinguishable mixtures in both the incoming and outgoing channels. This process enjoys no fermionic suppression and is described by the $\epsilon = 0$ case of Eq. (5.2).

5.3 Summary

We have found the measured dipolar relaxation rates in bosonic ^{162}Dy and fermionic ^{161}Dy to be in excellent agreement with rates predicted under the first-order Born

approximation. In particular, the scattering rates increase for bosons and decrease for fermions in the large B , low T limit. This confirmation of universal inelastic dipolar scattering demonstrates that dipolar relaxation in highly dipolar fermions is far less severe than in highly dipolar bosons, and dipolar relaxation will be less of a hindrance to experiments using large-spin fermions in studies of quantum many-body physics involving spin mixtures.

Chapter 6

Spin-orbit coupling with dysprosium

Topologically nontrivial materials require the coupling of charge carriers' momentum (or orbit) to their quantum-mechanical spin. This spin-orbit coupling (SOC) arises from electron movement through the intrinsic electric field of the crystal and can result in topological insulators and superconductors as well as exotic quantum Hall states [56, 57].

Ultracold neutral atoms can experience an analogous coupling in the presence of light fields that couple Zeeman sublevels via a two-photon Raman transition [58]. Recoil momentum is transferred as the optically coupled spin flips [59–61]. The adiabatic evolution of these dressed states as the atom moves in the Raman field creates a synthetic gauge field, which takes the form of SOC when these states form a degenerate manifold [62].

Spin-orbit coupling has been created in this manner in Bose-Einstein condensates

(BECs) [58, 63–70] and degenerate Fermi gases (DFGs) [19, 20, 71–73] of alkali-metals. However, a consequence of employing Raman coupling to generate SOC with alkali atoms is atomic heating due to spontaneous emission. This heating leads to loss of quantum degeneracy and trap population and severely limits the lifetimes of fermionic alkali gases [19, 20, 71], hampering the study of quantum many-body phenomena manifest at longer timescales.

Spontaneous emission can be eliminated while still producing large Raman coupling by using atoms with ground state orbital angular momentum $L > 0$, such as dysprosium. Large L 's ensure that the vector and tensor polarizabilities which give rise to Raman coupling matrix elements always scale as inverse atomic detuning Δ_a^{-1} [74, 75]. Because spontaneous emission decreases rapidly with detuning as Δ_a^{-2} , one can always choose a detuning that provides a large Raman coupling Ω_R while minimizing heating from incoherent scattering [21]. The large spin of dysprosium— $F = 21/2$ for ^{161}Dy —may also enable the study of (possibly non-Abelian) spinors coupled to large synthetic gauge fields [21, 76].

Though spontaneous emission can be eliminated, the strong dipole-dipole interaction (DDI) between highly magnetic, large-spin atoms induces decay from metastable Zeeman levels that leads to heating and trap loss [39, 40, 52]. However, by taking advantage of the fermionic suppression of dipolar relaxation discussed in Ch. 5, we are able to produce SOC gases of ^{161}Dy with lifetimes as long as 400 ms, surpassing that of ^{40}K and ^6Li fermionic SOC systems by factors of ~ 10 and 100, respectively [19, 20, 71], and comparable to the lifetime of bosonic SOC alkali gases [58].

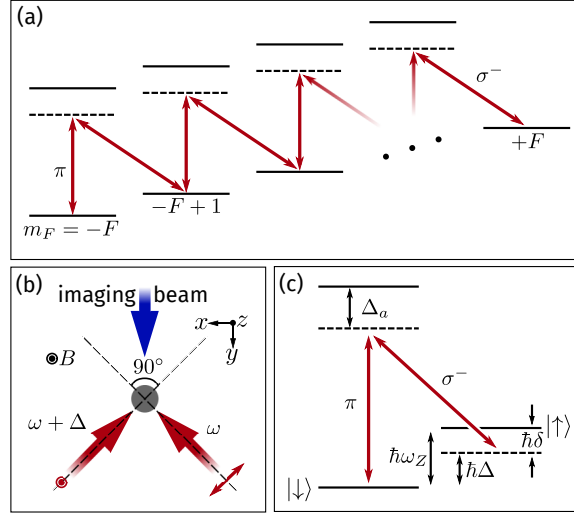


Figure 6.1: (a) Raman coupling with π and σ^- transitions for a large F atom with zero or small quadratic Zeeman shift. (b) Schematic of Raman beam geometry used to produce SOC with ^{161}Dy . (c) Coupling scheme for ^{161}Dy at a magnetic field high enough that with the lowest two Zeeman states are isolated by the quadratic Zeeman shift. The states that form this pseudospin-1/2 system are labeled $|\downarrow\rangle = |m_F = -21/2\rangle$ and $|\uparrow\rangle = |m_F = -19/2\rangle$.

6.1 Raman-coupling field configuration

We measured the effects of both elastic and inelastic dipolar collisions on gases of fermionic ^{161}Dy under SOC using the Raman coupling scheme sketched in Fig. 6.1(b)-(c). The lowest two Zeeman sublevels of the lowest ground state hyperfine manifold, $|\downarrow\rangle \equiv |F = 21/2, m_F = -21/2\rangle$ and $|\uparrow\rangle \equiv |21/2, -19/2\rangle$, were coupled by the two Raman lasers with wavelength $\lambda = 741$ nm and coupling strength Ω_R . The Raman lasers were detuned by several GHz from the 1.78(2)-kHz-wide 741-nm transition in ^{161}Dy [27]. A full schematic of the Raman lasers is shown in Fig. 4.2.

The first Raman beam propagated along $(\hat{x} - \hat{y})$ with frequency ω and was linearly polarized along $\hat{x} + \hat{y}$; i.e., it drove $\sigma^\pm$ transitions, though σ^+ are off resonant and not

shown in Fig. 6.1. The second Raman beam propagated along $-(\hat{x} + \hat{y})$ with frequency $\omega + \Delta$ and was linearly polarized along $\hat{z}$; i.e., it drove π transitions (the magnetic field is along $\hat{z}$). The detuning from two-photon Raman resonance is defined as $\delta = \omega_Z - \Delta$, where $\hbar\omega_Z$ is the Zeeman energy. Each Raman transition transfers $2\hbar k_R$ of momentum between an atom and the lasers, where $k_R = k_r \sin(\theta/2)$, $k_r = 2\pi/\lambda$ is the single-photon recoil momentum, and $\theta = 90^\circ$ is the angle between beams. The natural unit of energy is $E_R = (\hbar k_R)^2/2m$, where m is the mass.

We applied a homogeneous bias field of $B_0 = 33.846(5)$ G along $\hat{z}$, producing a Zeeman splitting $\hbar\omega_Z$. The quadratic Zeeman shift at this field is sufficient to produce a two-photon detuning between $|m_F = -19/2\rangle$ and $|m_F = -17/2\rangle$ of more than $70E_R/\hbar$. This allows us to neglect all Raman coupling to spin states with $m_F > -19/2$ when the Raman fields are on resonance with $|m_F = -21/2\rangle$ and $|m_F = -19/2\rangle$, i.e., when $\delta = 0$. Indeed, we observed (via Stern-Gerlach measurement) population in only these two states. Restricting to a pseudospin-1/2 system facilitates fermionic suppression of dipolar relaxation.

6.2 High-field degenerate Fermi gas

The DFG of ^{161}Dy was produced as described in Ch. 2. However, the dense Feshbach resonance spectrum of ^{161}Dy results in atom loss when sweeping to higher magnetic field [23]. Nevertheless, a high-field (~ 34 G) DFG of $1.7(3) \times 10^4$ atoms at $T/T_F = 0.44(1)$ was obtained and is shown in Fig. 6.2(c)-(d). The increase in T/T_F compared to the low-field DFG was due to both atom loss ($T_F \propto N^{1/3}$) and heating from anti-evaporation—colder, denser atoms are more likely to be lost by three-body collisions [37].

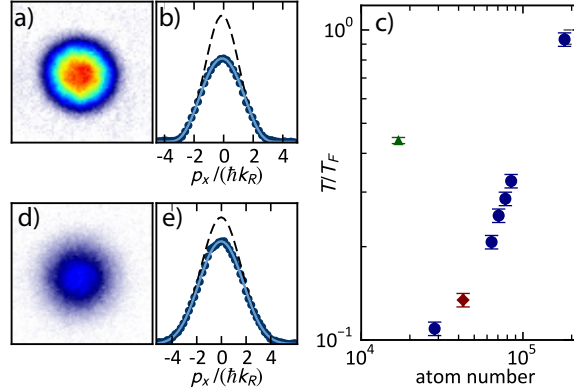


Figure 6.2: (a) DFG of ^{161}Dy at 0.48(1) G with $N = 4.3(2) \times 10^4$ atoms and $T/T_F = 0.13(1)$. (b) 1D integrated momentum distribution of DFG in (a) with Thomas-Fermi fit (solid line) and thermal fit to the wings (dashed line). (c) Single-species evaporation trajectory of ^{161}Dy . The red diamond corresponds to the DFG shown in (a). The green triangle corresponds to the DFG shown in (d). (d) DFG of ^{161}Dy at $B_0 = 33.846(5)$ with $N = 1.7(3) \times 10^4$ and $T/T_F = 0.44(1)$. (e) Same as in (b) but for DFG in (d).

The dense Feshbach resonance landscape of lanthanide atoms (see Ch. 3) requires special consideration when choosing a magnetic field at which to implement SOC. The magnetic field B_0 was chosen to both minimize three-body loss for spin-polarized in $|\downarrow\rangle$ and $|\uparrow\rangle$ states separately and to minimize loss in superpositions of these states: We verified through a measurement of the atom loss spectrum of mixed $|\downarrow\rangle$ and $|\uparrow\rangle$ states that this field sits in a region of near-minimal loss. We also observed that the features in Fig. 6.3(a) exhibit no dependence on coupling strength, unlike that found for alkali-atom gases under Raman coupling [63, 72, 73]. In those experiments, low-order single partial-wave Feshbach resonances were modified by SOC to induce higher-order effective partial-wave contributions. In dysprosium, however, many partial waves already contribute to Feshbach resonances [22], possibly rendering the SOC modification negligible.

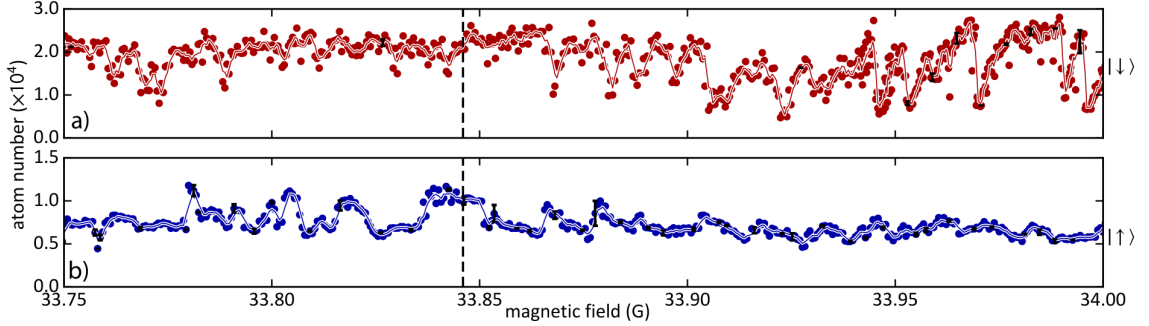


Figure 6.3: Atom loss spectroscopy of Feshbach resonances for ^{161}Dy in (a) $|\downarrow\rangle \equiv |m_F = -21/2\rangle$; (b) $|\uparrow\rangle \equiv |m_F = -19/2\rangle$. Both spectra were taken with the same initial atom number; note the different y -axis scales. The vertical dashed line indicates the field used to produce SOC fields, $B_0 = 33.846(5)$ G, and the solid lines are guides to the eye. The black error bars represent statistical errors on the subset of points indicated.

6.3 Spin-orbit coupling measurement

The Raman coupling is described by the effective single-particle Hamiltonian

$$\begin{aligned} \hat{H}_R &= \frac{[\hat{p}_x - 2\hbar k_R(\hat{F}_z + F)]^2}{2m} - \hbar\delta\hat{F}_z - \frac{\hbar\Omega_R}{2}(\hat{\Lambda} + \hat{\Lambda}^\dagger), \\ \hat{\Lambda} &= \hat{F}_+ \left(\hat{\mathbb{I}} - \frac{2\hat{F}_z + \hat{\mathbb{I}}}{2F + 3} \right) \left[\frac{2F + 3}{\sqrt{2F(4F + 2)}} \right], \end{aligned} \quad (6.1)$$

for the bare atomic states $|\downarrow, p_x\rangle$ and $|\uparrow, p_x + 2\hbar k_R\rangle$, where $\hat{F}_z$ is the spin-projection operator, $\hat{F}_+$ is the raising operator, $\hat{\mathbb{I}}$ is the identity, and $\hat{p}_x$ is the quasimomentum. This Hamiltonian contains equal strengths of Rashba and Dresselhaus SOC [21, 77, 78].

The gas was loaded into the lowest SOC-dressed band by adiabatically turning on the Raman coupling over a time period of 35 ms. We held the gas in this configuration for a time τ_{soc} . By diabatically turning off the coupling lasers, the quasimomentum

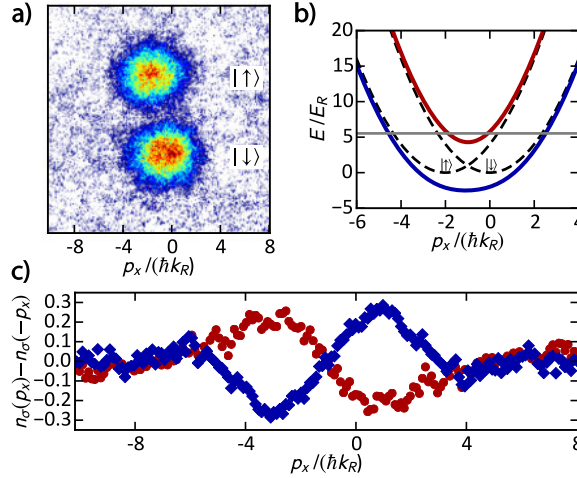


Figure 6.4: (a) Momentum distribution of a SOC-gas of ^{161}Dy with $\Omega_R = 5.9(2)E_R$ and $\delta = 0.1(2)E_R$ after diabatically removing the coupling and separating the spin states with a magnetic field gradient. Time-of-flight duration is 10 ms. (b) SOC dispersion curve for the gas in panel (a). The solid horizontal line indicates the Fermi energy for a gas of $N = 1.2 \times 10^4$ atoms in a trap with $\bar{f} = 150$ Hz. (c) Integrated momentum asymmetry along x for $\sigma = \downarrow$ (blue diamonds) and $\sigma = \uparrow$ (red circles).

of the dressed band is converted into mechanical momentum, which can be measured via time-of-flight imaging [79, 80]. A gas with $\Omega_R = 5.9(2)E_R$ is shown in Fig. 6.4(a) with the spin states separated in a Stern-Gerlach measurement. The corresponding quasimomentum dispersion is shown in Fig. 6.4(b). The SOC manifests itself as a momentum asymmetry in the bare spin states, i.e., $n_\sigma(p_x) \neq n_\sigma(-p_x)$ for $\sigma = \{\downarrow, \uparrow\}$ [19]. This momentum asymmetry is highlighted in Fig. 6.4(c) by plotting the reflected momentum difference of the two spin states, $n_\sigma(p_x) - n_\sigma(-p_x)$.

We were unable to perform spin-injection or momentum-resolved radio-frequency spectroscopy, as has been done in the fermionic SOC ^6Li and ^{40}K systems [19, 20]. The relatively large mass of ^{161}Dy means that to achieve a similar band structure resolution, field stability would have to be less than a few hundreds of μG , a factor of ten lower than our capability.

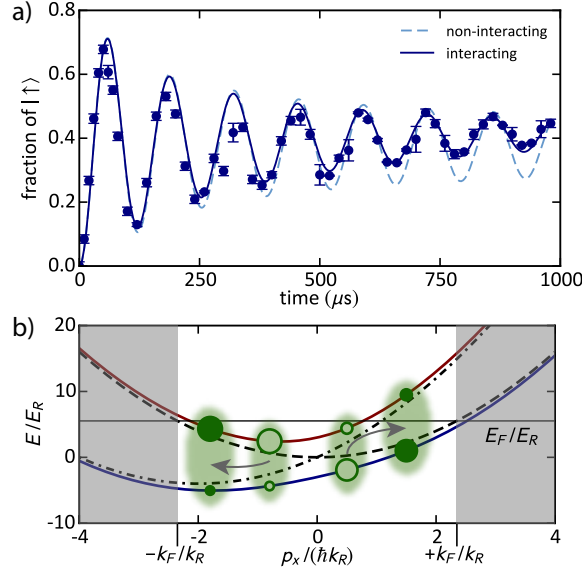


Figure 6.5: (a) Rabi oscillation data with $\Omega_R = 6.8(2)E_R$ and $\delta = -3.8(2)E_R$. Predicted spin dynamics for a non-interacting Fermi gas (dashed line) according to Eq. 6.2 and an interacting Fermi gas (solid line) according to a Monte-Carlo simulation. (b) Cartoon illustrating elastic collisions during Rabi oscillations. The dashed (dot-dashed) line is the uncoupled $|\downarrow\rangle$ ($|\uparrow\rangle$) quasimomentum dispersion at B_0 . The solid colored quasimomentum dispersions are for the dressed states with $\Omega_R = 6E_R$ and $\delta = -4E_R$. The Fermi energy is marked by the solid horizontal line, and an uncoupled $|\downarrow\rangle$ -polarized Fermi gas at $T = 0$ occupies the quasimomenta between $\pm k_F$ (unshaded region). See text for explanation.

6.4 Elastic dipolar collisions in Rabi oscillations

In order to calibrate the strength of the Raman coupling, we drove Rabi oscillations by diabatically turning on the coupling lasers in $<5 \mu s$. For a non-interacting system, the probability of observing an atom in state $|\uparrow\rangle$ after being initialized in state $|\downarrow\rangle$ is [19]

$$P_{\uparrow}(k_x, t) = \frac{\sin^2 \left(\sqrt{\left(\frac{\hbar k_x k_R}{m} + \frac{\delta}{2} \right)^2 + \left(\frac{\Omega_R}{2} \right)^2} t \right)}{1 + \left(\frac{2\hbar k_x k_R}{\Omega_R m} + \frac{\delta}{\Omega_R} \right)^2}, \quad (6.2)$$

where $\hbar k_x = p_x$ and t is the duration of the Raman pulse. The spread of k_x in the DFG causes damping of the total spin dynamics due to Doppler shifts. By integrating Eq. (6.2) over the measured momentum distribution of the DFG, the Rabi spin dynamics of a non-interacting DFG are obtained as plotted in Fig. 6.5(a).

However, elastic collisions can change an atom's momentum along the SOC dimension $\hat{x}$, which modifies the spin dynamics and results in additional Rabi-oscillation dephasing. This process is illustrated in Fig. 6.5(b). The atoms begin on the $|\downarrow\rangle$ dispersion but project onto a superposition of the two dressed dispersions when the Raman-coupling diabatically turns on. We consider the two atoms shown as green shaded ovals at the center of the figure as examples. Each oval covers green circles that depict the $|\downarrow\rangle$ -components of the dressed state belonging to a single atom with the size of each circle proportional to the $t = 0$ projection of dressed state onto the bare $|\downarrow\rangle$ state at the given quasimomentum. The atoms begin to Rabi oscillate between $|\downarrow\rangle$ and $|\uparrow\rangle$. The Raman coupling resonant is only resonant at the degeneracy point $p_x = 0\hbar k_R$ where the Rabi oscillations exhibit full contrast. Atoms at all other quasimomenta have an effective detuning (i.e., a Doppler shift) that reduces the Rabi oscillation contrast, as is the case for the data in Fig. 6.5(a).

When elastic collisions between two atoms can change both atoms' momenta along the SOC axis, the effective Rabi detunings of each atom also changes. For example, the atoms represented by the hollow green circles could elastically scatter to states represented by solid green circles while conserving total momentum and energy (possibly by exchanging momentum along other dimensions). The subsequent spin dynamics depart from the pre-collision Rabi oscillation, resulting in dephasing and a higher steady-state occupation of $|\uparrow\rangle$ not observed in Eq. (6.2).

While collisional effects were negligible in previous SOC fermionic alkali gases [19, 20], elastic dipolar collisions due to ^{161}Dy 's large dipole moment (10 Bohr magnetons) are not negligible on the timescale of Ω_R^{-1} in our system. In order to account for momentum changing collisions, we use a Monte Carlo simulation that includes elastic collisions between pairs of atoms. Each simulated atom is assigned an initial momentum sampled from the Fermi-Dirac momentum distribution and has a spinor of the form

$$|\psi(t)\rangle = c_{\uparrow}(t)|\uparrow\rangle + c_{\downarrow}(t)|\downarrow\rangle, \quad (6.3)$$

initialized to $c_{\downarrow}(0) = 1$ and $c_{\uparrow}(0) = 0$. The spinor of each atom then evolves according to

$$c_{\downarrow,\uparrow}(t) = \left\{ c_{\downarrow,\uparrow}(0) \cos(t\Theta/2) \right. \\ \left. \pm i \left[\frac{\delta}{\Theta} c_{\downarrow,\uparrow}(0) \mp \frac{\Omega_R}{\Theta} c_{\uparrow,\downarrow}(0) + \frac{2\hbar}{m\Theta} k_R(k_R + k_x) c_{\downarrow,\uparrow}(0) \right] \sin(t\Theta/2) \right\} e^{-i\omega t}, \quad (6.4)$$

$$\omega = \frac{\hbar(2k_R^2 + 2k_R k_x + k_x^2)}{2m},$$

$$\text{and} \quad \Theta = \sqrt{4 \left[\frac{\hbar k_R(k_R + k_x)}{m} \right]^2 + 4\delta \left[\frac{\hbar k_R(k_R + k_x)}{m} \right] + \delta^2 + \Omega_R^2}.$$

For two atoms colliding at time $t = t'$, the spinor of each atom does not change immediately, i.e., $c_{\downarrow}(t'^-) = c_{\downarrow}(t'^+)$ and $c_{\uparrow}(t'^-) = c_{\uparrow}(t'^+)$ for both atoms. However, the collision does change the momentum of each particle, i.e., $k_x(t'^-) \neq k_x(t'^+)$, and thus changes the subsequent evolution of the spinors through Eq. (6.4). This results in not only more dephasing but also an equal steady-state population of $|\uparrow\rangle$ and $|\downarrow\rangle$

because the collisions effectively erase the effect of the initial conditions $c_{\downarrow}(0) = 1$ and $c_{\uparrow}(0) = 0$. Averages over all simulated atomic spinors produce the solid-line Rabi-oscillation curve in Fig. 6.5(a).

When fitting the data, the rate of collisions and Raman coupling strength were treated as free parameters. Though the simple Monte Carlo model does not take into account correlations involving the atoms' positions or other details of the collisions, the model does capture the increased damping of the oscillations and higher steady state of fraction of $|\uparrow\rangle$ atoms.

Curves from both the Monte Carlo simulation and the non-interacting case of Eq. (6.2) are shown with data in Fig. 6.5(a). The fitted elastic collision rate of $9.9(2) \mu\text{s}^{-1}$ was larger than the expected rate from dipolar collisions, $3(1) \mu\text{s}^{-1}$ ¹. However, a more complete simulation, perhaps using the direct simulation Monte Carlo technique (DSMC) of Ref. [45], may yield a more accurate collision rate by taking into account all correlations between position, spin, and momentum.

These measurements indicate a weakly interacting Fermi gas under the influence of the Raman-coupling necessary to generate synthetic gauge fields. Such a gas could complement lattice-bound fermionic polar molecule systems in which dipolar interactions can also influence spin dynamics [81].

¹We note that the particular fitted value of the collision rate has negligible effect on the fitted value of $\Omega_R = 6.57(4)E_R$; the covariance is small, 7×10^{-4} .

6.5 Dipolar relaxation in the SOC dressed state

In addition to elastic collisions, the DDI also induces two-body inelastic collisions (dipolar relaxation), which can cause heating or atom loss. When the atoms are adiabatically loaded into the SOC dressed states of Eq. (6.1), the spinor is a superposition of the bare spin states. The $|\uparrow\rangle$ portion of an atom's spin-state can undergo dipolar relaxation even when in the lowest dressed band shown in blue in Fig. 6.4(b). This is especially significant if the Zeeman energy is larger than twice the trap depth because a single spin flip releases sufficient energy to eject both atoms from the trap, as is the case for fields above ~ 50 mG in our optical dipole trap.

6.5.1 Measurement of SOC dressed state decay

We measured the dipolar relaxation out of the SOC-dressed states by varying the hold time τ_{soc} in the adiabatically loaded dressed states. After turning off the coupling and releasing the atoms from the trap, we measured the atom number via absorption imaging and fit the atom loss curve to a numerically integrated rate equation as in Ch. 5. Terms for both one-body loss β_1 (fixed parameter) and two-body loss β_2 (free parameter) were included:

$$\frac{dN}{dt} = -\beta_1 N - \beta_2 \bar{V}^{-1} N^2, \quad (6.5)$$

where $\bar{V} = \sqrt{8}(2\pi)^{3/2}\sigma_x\sigma_y\sigma_z$ is the mean collisional volume for a harmonically trapped thermal gas of Gaussian width σ_i . We fixed β_1 to the background scattering rate $(20 \text{ s})^{-1}$. No heating due to the Raman lasers was measured, confirming our expectation that this $L = 6$ atom would be immune to spontaneous emission under SOC [21].

The fitted values of β_2 for different coupling strengths and $\delta = 0.0(3)E_R$ are shown in Fig. 6.6(a). These may be compared with the dipolar relaxation rates found in Ch. 5 for non-SOC Dy gases at 1 G: $[0.3, 10, 30] \times 10^{-12} \text{ cm}^3\text{s}^{-1}$ for identical fermions, distinguishable particles, and identical bosons, respectively.

Because two-body loss is density dependent, the characteristic timescale τ_2 can only be defined by multiplying the loss parameter by a density: $\tau_2 = (\bar{n}(0)\beta_2)^{-1}$, where $\bar{n}(0) = N/\bar{V}$ was a typical initial density for our system. Alternatively, one may care most about preserving interactions in the system and therefore define the relevant experimental timescale as the time required for the ratio of the dipolar energy to Fermi energy $\epsilon(t) = E_{dd}(t)/E_F(t)$ to decrease to $\epsilon(\tau_{exp}) = \epsilon(t=0)/e$, where $E_{dd} = N\mu_0\mu^2(48\pi^{3/2}\sigma_x\sigma_y\sigma_z)^{-1}$ [45] and $E_F = \hbar\bar{f}(6N)^{1/3}$. The values of τ_{exp} and τ_2 shown in Fig. 6.6(b) are for a gas of $N = 1 \times 10^4$ atoms at $T/T_F = 0.4$ and $\bar{f} = 150 \text{ Hz}$. For such a gas, $\epsilon(0) = 0.56\%$ and $\bar{n}(0) = 1.2 \times 10^{13} \text{ cm}^{-3}$.

To test the importance of fermionic suppression in achieving long lifetimes, we also measured the ^{161}Dy lifetime at low-field 0.48(1) G with $\Omega_R = 5.9(3)E_R$ and $\delta = -0.1(2)$. At this low magnetic field, the quadratic Zeeman shift is insufficient to limit Raman coupling to only the $|m_F = -21/2\rangle$ and $|m_F = -19/2\rangle$ states, so most collisions involve many m_F states and are distinguishable. Furthermore, any indistinguishable collisions have less suppression due to the lower Zeeman energy released. Indeed, the measured low-field two-body loss parameter at this field was $\beta_2 = 2.1(4) \times 10^{-11} \text{ cm}^3/\text{s}$. This is $\sim 16\times$ larger than the high-field two-body loss parameter $\beta_2 = 1.28(2) \times 10^{-12} \text{ cm}^3/\text{s}$ for the same Ω_R at B_0 . The corresponding low-field characteristic timescale at density $\bar{n} = 1.2 \times 10^{13} \text{ cm}^{-3}$ is only $\tau_2 = 4.0(8) \text{ ms}$.

We also measured similarly short—<10-ms—lifetimes for Raman-dressed bosonic

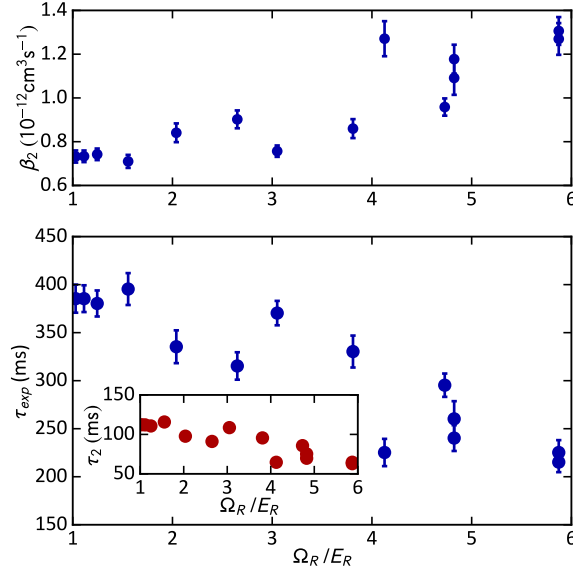


Figure 6.6: (a) Two-body loss parameters for $\delta = 0.0(3)E_R$. (b) Experimental lifetimes for a gas of 1×10^4 ^{161}Dy atoms with $T/T_F = 0.4$ and $\bar{f} = 150$ Hz, where the ratio of the dipolar and Fermi energies fall by e in time τ_{exp} . (inset) Characteristic lifetimes $\tau_2 = (\bar{n}(0)\beta_2)^{-1}$ with $\bar{n}(0) = 1.2 \times 10^{13} \text{cm}^{-3}$.

^{162}Dy , even at low field where β_2 should be minimized for distinguishable particles and Bosons. These results imply that proposals for observing exotic non-Abelian spinors and many-body physics using SOC dipolar BECs will be quite difficult to realize [21, 82–85].

6.5.2 First-order Born approximation

We calculate dipolar relaxation rates in the SOC dipolar Fermi gas using an approach that is similar to that found in Refs. [86, 87] for calculating loss from metastable dressed bands in a non-dipolar gas. We consider scattering from the two-atom state $|\gamma\rangle = |\mathbf{p}_1\rangle|\alpha_1(\mathbf{p}_1)\rangle|\mathbf{p}_2\rangle|\alpha_2(\mathbf{p}_2)\rangle$, where $|\alpha_i(\mathbf{p}_i)\rangle$ is the spinor in the dressed band α_i and $\mathbf{p}_i$ is the quasimomentum of atom i , to the final state $|\gamma'\rangle$, which exists in

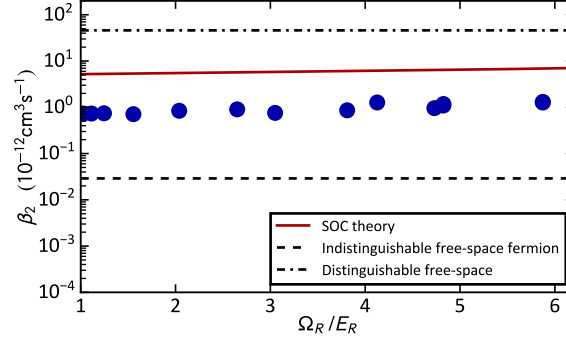


Figure 6.7: Measured and theoretical two-body loss parameters at B_0 with $\delta = 0.0(3)E_R$. The black dashed (dot-dashed) line is calculated for a gas of identical dipolar fermions (distinguishable particles) with spin mixtures under no SOC. The solid red line is calculated for SOC dipolar fermions under using Eq. (6.6).

a lower dressed-state manifold $\mathcal{M}$ [88] labeled by the imbalance M between the number of photons in one Raman-beam versus the other. The two-body loss parameter is defined as

$$\beta_2 = \frac{8}{m^2} \sum_{\alpha'_1, \alpha'_2} \int d\mathbf{p}_1 d\mathbf{p}_2 d\mathbf{p}'_1 d\mathbf{p}'_2 \delta_E \delta_{\mathbf{p}'} \times \Pi(\mathbf{p}_1) \Pi(\mathbf{p}_2) |f(\gamma', \gamma)|^2, \quad (6.6)$$

where $f(\gamma', \gamma)$ is the scattering amplitude between γ and γ' ; $\Pi(\mathbf{p}_i)$ is the initial quasimomentum distribution of particle i ; and δ_E and $\delta_{\mathbf{p}'}$ ensure conservation of energy and momentum, respectively. The summation over α_i includes all dressed bands, and we assumed the atoms are initially in the lowest band.

Because the gas is adiabatically loaded into the SOC-dressed band, the quasimomentum distribution is simply the Fermi-Dirac momentum distribution of the initial gas

$$\Pi(\mathbf{p}_i) = \frac{\text{Li}_{3/2}[-\zeta e^{-p_i^2/(2mk_B T)}]}{(2\pi mk_B T)^{3/2} \text{Li}_3[-\zeta]}, \quad (6.7)$$

where $\text{Li}_s[x]$ is the polylogarithm series and ζ is the fugacity of the DFG.

All that remains is to determine the dipolar relaxation scattering amplitude in the SOC frame, $f(\gamma', \gamma)$. We first recall the transformations necessary to produce the Hamiltonian in Eq. (6.1). In the lab frame, the single particle SOC Hamiltonian for particle i is

$$\hat{H}_i(t) = \frac{\hat{\mathbf{p}}_i^2}{2m} - \hbar\omega_Z \hat{F}_{z,i} - \frac{\hbar\Omega_R}{2} \left(e^{i(2k_R x_i - \Delta t)} \hat{\Lambda}_i + \text{H.c.} \right). \quad (6.8)$$

We apply two transformations $\hat{U}_{t,i} = e^{-i\Delta t \hat{F}_{z,i}}$ and $\hat{U}_{k,i} = e^{i2k_R x_i \hat{F}_{z,i}}$ to recover the form of Eq. (6.1) with unmodified parabolic dispersions along y and z . The SOC Hamiltonian for atom i is then

$$\hat{H}_i = \hat{H}_{R,i} + \frac{\hat{p}_{y,i}^2 + \hat{p}_{z,i}^2}{2m}. \quad (6.9)$$

This is extended to two atoms, as required for dipolar relaxation:

$$\hat{H}_{12} = \hat{H}_1 \otimes \hat{\mathbb{I}}_2 + \hat{\mathbb{I}}_1 \otimes \hat{H}_2, \quad (6.10)$$

where $\hat{\mathbb{I}}_i$ is the identity in the subspace of atom i .

The dipole-dipole interaction has the form

$$\hat{H}_{dd} = \frac{\mu_0 (g_F \mu_B)^2}{4\pi r^3} \left[\hat{\mathbf{F}}_1 \cdot \hat{\mathbf{F}}_2 - 3(\hat{\mathbf{F}}_1 \cdot \tilde{\mathbf{r}})(\hat{\mathbf{F}}_2 \cdot \tilde{\mathbf{r}}) \right], \quad (6.11)$$

where $\mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2$ and $\tilde{\mathbf{r}} = \mathbf{r}/|\mathbf{r}|$. Combining Eq. (6.10) and (6.11) into the full Hamiltonian yields

$$\hat{H} = \hat{H}_{12} + \hat{H}'_{dd}, \quad (6.12)$$

where

$$\hat{H}'_{dd} = \hat{U}_{k,1}^\dagger \hat{U}_{k,2}^\dagger \hat{U}_{t,2}^\dagger \hat{U}_{t,1}^\dagger \hat{H}_{dd} \hat{U}_{t,2} \hat{U}_{t,1} \hat{U}_{k,1} \hat{U}_{k,2}. \quad (6.13)$$

Although both single- and double-spin-flip inelastic processes can occur, we restrict our consideration to single-flip events. (Double-flip processes for $|\uparrow\rangle$ are slower by roughly a factor of $F = 21/2$ [40].) The single-spin-flip portion is

$$\hat{H}_{dd}^{(1)} \propto [\hat{F}_{z,1} \hat{F}_{-,2} + \hat{F}_{-,1} \hat{F}_{z,2}], \quad (6.14)$$

which in the SOC frame becomes

$$\hat{H}_{dd}^{(1)} \propto e^{i\Delta t} [e^{-i2k_R x_2} \hat{F}_{z,1} \hat{F}_{-,2} + e^{-i2k_R x_1} \hat{F}_{-,1} \hat{F}_{z,2}]. \quad (6.15)$$

With this Hamiltonian, the single-spin flip scattering amplitude under the first-order Born approximation in the SOC frame is

$$\begin{aligned} f^{(-1)}(\gamma', \gamma) = & \frac{m\mu_0(g_F\mu_B)^2}{16\pi\hbar^2} \sqrt{\frac{24}{5\pi}} e^{i\Delta t} \\ & \times \left\langle \gamma' \left| \frac{Y_{2,-1}^*(\hat{\mathbf{r}})}{r^3} \left[e^{-i2\hbar k_R x_2} \hat{F}_{z,1} \hat{F}_{-,2} + e^{-i2\hbar k_R x_1} \hat{F}_{-,1} \hat{F}_{z,2} \right] \right| \gamma \right\rangle, \end{aligned} \quad (6.16)$$

where $Y_{lm}(\hat{\mathbf{r}})$ is a spherical harmonic. We limit ourselves to the first-order Born approximation because our reproduction of the calculation in Ref. [86] for metastable dressed-band decay showed that higher order corrections in the Lippman-Schwinger equation only contributed a 10^{-4} level correction.

The modification of the inelastic DDI in the SOC rotating frame is due to the commutation relation of $\hat{F}_-$ and $\hat{F}_z$. The time dependent exponential does not affect the calculation of Eq. (6.6) because Eq. (6.16) only enters as the modulus

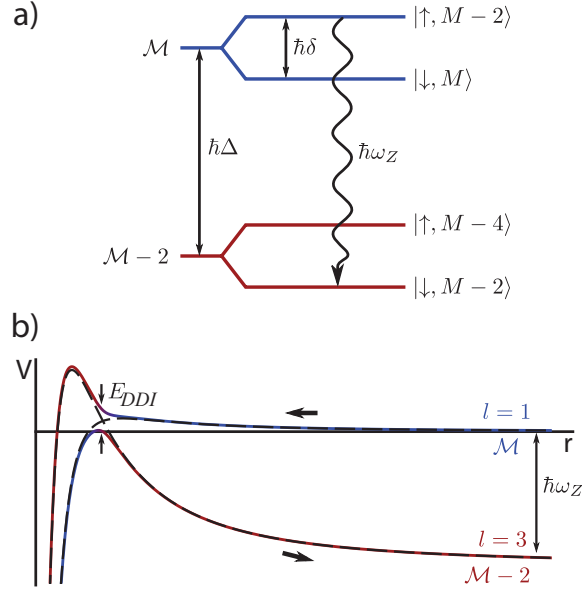


Figure 6.8: (a) A system initially in photon number imbalance manifold $\mathcal{M}$ can decay to manifold $\mathcal{M} - 2$ via the DDI, releasing the Zeeman energy $\hbar\omega_Z$ as kinetic energy. Each state is labeled by the atom's spin state and the Raman lasers' photon number imbalance. The DDI can only couple $|\uparrow\rangle$ to $|\downarrow\rangle$ and cannot change the photon imbalance. (b) Interatomic potentials for identical fermions entering a collision on the blue p -wave ($l = 1$) orbital potential and exiting on the red f -wave ($l = 3$) orbital potential. The potentials are shown in the rotating SOC frame and with $\omega_Z \gg \delta$. The DDI couples these two potentials with energy E_{DDI} . Atoms may exit on the lower dressed potential, releasing Zeeman energy $\hbar\omega_Z$. The uncoupled potentials are shown as black dashed lines.

squared. However, the position dependent exponentials change the quasimomentum conservation condition: Whereas conservation of momentum would typically require $\mathbf{p}_1 + \mathbf{p}_2 = \mathbf{p}'_1 + \mathbf{p}'_2$, the quasimomentum conservation condition is instead $\mathbf{p}_1 + \mathbf{p}_2 = \mathbf{p}'_1 + \mathbf{p}'_2 + 2\hbar k_R \hat{x}$. Thus, quasimomentum is conserved up to $2\hbar k_R$ for single-spin-flip dipolar relaxation in the SOC frame, which is due to the atom changing spin without exchanging a photon between the Raman laser fields.

This lack of photon exchange leads to an intuitive picture for the relaxation in

which the system decays from photon imbalance manifold $\mathcal{M}$ to manifold $\mathcal{M} - 2$, as illustrated in Fig. 6.8(a). Within each manifold the system state is labeled by the atom's spin state and the Raman lasers' photon imbalance. For an atom initially in $|\downarrow\rangle$ with photon imbalance M , the state can be written as

$$|\Phi_{\mathcal{M}}\rangle = \phi_{\downarrow}|\downarrow, M\rangle + \phi_{\uparrow}|\uparrow, M - 2\rangle, \quad (6.17)$$

because the atom's spin state is coherently changed by exchanging a photon from one laser to the other, changing the imbalance by 2. In contrast, if the DDI incoherently changes the $|\uparrow\rangle$ portion of Eq. (6.17) to $|\downarrow\rangle$, the atom's new dressed state can be written as

$$|\Phi'_{\mathcal{M}-2}\rangle = \phi'_{\downarrow}|\downarrow, M - 2\rangle + \phi'_{\uparrow}|\uparrow, M - 4\rangle. \quad (6.18)$$

The coefficients ϕ_{σ} (ϕ'_{σ}) depend on the initial (final) dressed band and momentum. The transition rate between Eq. (6.17) and (6.18) then depends on their overlap under dipolar relaxation,

$$\langle\Phi'_{\mathcal{M}-2}|H_{dd}|\Phi_{\mathcal{M}}\rangle = \phi_{\uparrow}\phi'_{\downarrow}\langle\downarrow, M - 2|H_{dd}|\uparrow, M - 2\rangle, \quad (6.19)$$

which results in a coupling energy E_{DDI} . Whereas the energy required to move a photon from one laser to the other compensates for the Zeeman energy difference (up to an amount δ) in a coherent process, the lack of photon exchange in dipolar relaxation causes the full Zeeman energy to be released into the system as kinetic energy when an atom's spin is flipped.

The relaxation process may also be viewed as a transition between two molecular potentials [89]. As illustrated in Fig. 6.8(b), identical fermions colliding on the p -wave

($l = 1$) potential can exit on an f -wave ($l = 3$) potential of different photon imbalance number $\mathcal{M}$, with a transition rate set by the DDI coupling. Fermionic suppression of dipolar relaxation occurs because inelastic dipolar collisions, unlike elastic dipolar collisions, are a short-range process [40, 52]: identical ultracold fermions must surmount the p -wave centrifugal barrier to collide, leading to a kinematic suppression.

6.6 Summary

While creating spin-orbit coupled fermionic alkali atoms using Raman coupling has been very successful, the short spontaneous-emission-limited lifetime of alkali atoms severely hampers investigations of interacting topologically non-trivial systems. We have demonstrated that spin-orbit-coupling in lanthanide atomic gases, specifically ^{161}Dy , can have much longer lifetimes that are instead limited by inelastic dipolar collisions. Fortunately, the dipolar relaxation is not rapid, allowing the study of spin-orbit coupled degenerate dipolar Fermi gases of Dy for as long as 400 ms, far longer than in fermionic alkali gases. Looking forward, the elastic dipolar collisions observed here may be exploited to study interacting spin-orbit coupled Fermi gases. Moreover, the lifetime of these Raman-coupled dipolar Fermi gases may now be sufficiently long to create and study exotic quantum liquids [90] as well as quantum Hall ribbons in synthetic dimensions [91–93] where the large Dy spin provides a wide bulk with which to isolate edge states.

Appendix A

Dipolar relaxation rates

This appendix contains examples of calculating specific dipolar relaxation rates. All possible output states must be accounted for, and special consideration must be given to the symmetries of the input and output states. We hope that the subtleties are illuminated through these examples.

The dipolar relaxation rates (Eq. 5.10) are calculated from the dipolar relaxation cross sections (Eq. 5.7 and 5.8). For convenience those equations are reproduced here:

$$\beta_{dr} = \left\langle \left[\sum_i \frac{g_n^i \sigma_1^i(k_{f1}/k_i, \epsilon)}{g_d^i} + \sum_j \frac{g_n^j \sigma_2^j(k_{f2}/k_i, \epsilon)}{g_d^j} \right] v_{\text{rel}} \right\rangle_{\text{thermal}}, \quad (\text{A.1})$$

$$\sigma_1 = \left| \frac{1}{2} \langle \text{out} | (F_{1z} F_{2-} + F_{1-} F_{2z}) | \text{in} \rangle \right|^2 \sigma(k_{f1}/k_i, \epsilon), \quad (\text{A.2})$$

$$\sigma_2 = \left| \frac{1}{2} \langle \text{out} | F_{1-} F_{2-} | \text{in} \rangle \right|^2 \sigma(k_{f2}/k_i, \epsilon), \quad (\text{A.3})$$

where

$$\sigma(k_{f\eta}/k_i, \epsilon) = \frac{8\pi}{15} \left(\frac{\mu_0 (g_F \mu_B)^2 m}{4\pi \hbar^2} \right)^2 [1 + \epsilon h(k_{f\eta}/k_i)] \frac{k_{f\eta}}{k_i},$$

and

$$(\hbar k_{f\eta})^2 = (\hbar k_i)^2 + \eta \times (2\mu\Delta E).$$

Note that the final momenta for the single- and double-spin-flip processes are different due to conservation of energy and $\sigma(k_{f2}/k_i, \epsilon) \neq \sigma(k_{f1}/k_i, \epsilon)$.

For initial spin mixtures (e.g., species A and B), the coupled rate equations for population loss (Eq. 5.11 in the Ch. 5) involve four loss rates. The two intraspecies loss rates are $\beta_{dr}^{A;A}$ and $\beta_{dr}^{B;B}$. The two interspecies loss rates are treated as distinct but are equivalent, $\beta_{dr}^{A;B} = \beta_{dr}^{B;A}$.

In the examples that follow, the two-atom spinors $|\text{in}\rangle$ and $|\text{out}\rangle$ are written in the form $|m_F\rangle|m_F\rangle$. The first ket refers to atom 1; the second to atom 2. A subscript $\mathcal{S}$ denotes a symmetrized state, i.e.,

$$|A\rangle|B\rangle_{\mathcal{S}} = \frac{1}{\sqrt{2}}(|A\rangle|B\rangle + |B\rangle|A\rangle).$$

We consider the atoms to be fermions (specifically ^{161}Dy with $F = 21/2$), so $\epsilon = -1$ or 0 for indistinguishable or distinguishable statistics, respectively. This is tracked in the factor $\sigma(k_{f\eta}/k_i, \epsilon)$. Notably, $\epsilon = -1$ for all cross sections here except for A.26 and A.27.

A.1 Spin-polarized gas, $|m_F = +21/2\rangle$

In a spin-polarized gas of $|m_F = +21/2\rangle$, the only incoming two-atom spinor is $|\text{in}\rangle = |21/2\rangle|21/2\rangle$. For such a state, two outgoing relaxation channels exist:

$$|\text{out}\rangle = |21/2\rangle|19/2\rangle_{\mathcal{S}}, \quad \text{single-spin-flip}; \quad (\text{A.4})$$

$$|\text{out}\rangle = |19/2\rangle|19/2\rangle, \quad \text{double-spin-flip}. \quad (\text{A.5})$$

The cross section for channel A.4 from Eq. A.2 is

$$\begin{aligned} \sigma_{(\text{A.4})} &= \left| \frac{1}{2} \langle 21/2 | \langle 19/2 |_{\mathcal{S}} (F_{1z}F_{2-} + F_{1-}F_{2z}) | 21/2 \rangle | 21/2 \rangle \right|^2 \sigma(k_{f1}/k_i, -1) \\ &= \left| \frac{21}{2} \sqrt{\frac{21}{2}} \right|^2 \sigma(k_{f1}/k_i, -1) = \left(\frac{21}{2} \right)^3 \sigma(k_{f1}/k_i, -1). \end{aligned} \quad (\text{A.6})$$

The cross section for channel A.5 from Eq. A.3 is

$$\begin{aligned} \sigma_{(\text{A.5})} &= \left| \frac{1}{2} \langle 19/2 | \langle 19/2 | F_{1-}F_{2-} | 21/2 \rangle | 21/2 \rangle \right|^2 \sigma(k_{f2}/k_i, -1) \\ &= \left(\frac{21}{2} \right)^2 \sigma(k_{f2}/k_i, -1). \end{aligned} \quad (\text{A.7})$$

Both collisions involve identical spin states in the incoming channel so $g_d = 2$. Both collisions remove two atoms initially in $m_F = 21/2$ from the trap so $g_n = 2$. Combining Eq. A.6, A.7, and A.1, the total collisional loss rate from the $m_F = 21/2$ state is

$$\beta_{dr}^{21/2;21/2} = \left\langle \left[\left(\frac{21}{2} \right)^3 \sigma(k_{f1}/k_i, -1) + \left(\frac{21}{2} \right)^2 \sigma(k_{f2}/k_i, -1) \right] v_{\text{rel}} \right\rangle_{\text{thermal}}. \quad (\text{A.8})$$

A.2 Spin mixture, $|m_F = -19/2\rangle$ & $|m_F = -21/2\rangle$

For a spin mixture of $|-19/2\rangle$ and $|-21/2\rangle$, we must consider two different incoming two-atom spinors:

$$|\text{in}\rangle = |-19/2\rangle|-21/2\rangle, \quad \text{mixture collision;} \quad (\text{A.9})$$

$$|\text{in}\rangle = |-19/2\rangle|-19/2\rangle, \quad \text{polarized collision.} \quad (\text{A.10})$$

One may mistakenly think that input state should be actually be the symmetric state $|\text{in}\rangle = |-19/2\rangle|-21/2\rangle_S$. However, for the incoherent spin mixture considered here, each atom's initial state is distinct.

A.2.1 $|\text{in}\rangle = |m_F = -19/2\rangle|m_F = -21/2\rangle$

For the initial state $|\text{in}\rangle = |-19/2\rangle|-21/2\rangle$, one outgoing relaxation channel exists:

$$|\text{out}\rangle = |-21/2\rangle|-21/2\rangle, \quad \text{single-spin-flip.} \quad (\text{A.11})$$

The cross section for channel A.11 from Eq. A.2 is

$$\begin{aligned} \sigma_{(\text{A.11})} &= \left| \frac{1}{2} \langle -21/2 | \langle -21/2 | (F_{1z}F_{2-} + F_{1-}F_{2z}) | -19/2 \rangle | -21/2 \rangle \right|^2 \sigma(k_{f1}/k_i, -1) \\ &= \left| \frac{1}{\sqrt{2}} \frac{21}{2} \sqrt{\frac{21}{2}} \right|^2 \sigma(k_{f1}/k_i, -1) = \frac{1}{2} \left(\frac{21}{2} \right)^3 \sigma(k_{f1}/k_i, -1). \end{aligned} \quad (\text{A.12})$$

This collision does not involve identical spin states in the incoming channel so $g_d = 1$. This collision removes one atom initially in $m_F = -21/2$ and a second atom initially in $m_F = -19/2$ from the trap; therefore, there are two population collisional

loss rates, both with $g_n = 1$. Combining Eq. A.12 and A.1, the total collisional loss rates from the $m_F = -21/2$ state and $m_F = -19/2$ states are

$$\beta_{dr}^{-21/2;-19/2} = \beta_{dr}^{-19/2;-21/2} = \left\langle \left[\frac{1}{2} \left(\frac{21}{2} \right)^3 \sigma(k_{f1}/k_i, -1) \right] v_{\text{rel}} \right\rangle_{\text{thermal}}. \quad (\text{A.13})$$

A.2.2 $|\text{in}\rangle = |m_F = -19/2\rangle |m_F = -19/2\rangle$

For the initial state $|\text{in}\rangle = |-19/2\rangle |-19/2\rangle$, two outgoing relaxation channels exist:

$$|\text{out}\rangle = |-21/2\rangle |-19/2\rangle_S, \quad \text{single-spin-flip}; \quad (\text{A.14})$$

$$|\text{out}\rangle = |-21/2\rangle |-21/2\rangle, \quad \text{double-spin-flip}. \quad (\text{A.15})$$

The cross section for channel A.14 from Eq. A.2 is

$$\begin{aligned} \sigma_{(\text{A.14})} &= \left| \frac{1}{2} \langle -21/2 | \langle -19/2 |_S (F_{1z} F_{2-} + F_{1-} F_{2z}) | -19/2 \rangle | -19/2 \rangle \right|^2 \sigma(k_{f1}/k_i, -1) \\ &= \left| -\frac{19}{2} \sqrt{\frac{21}{2}} \right|^2 \sigma(k_{f1}/k_i, -1) = \frac{21}{2} \left(\frac{19}{2} \right)^2 \sigma(k_{f1}/k_i, -1). \end{aligned} \quad (\text{A.16})$$

The cross section for channel A.15 from Eq. A.3 is

$$\begin{aligned} \sigma_{(\text{A.15})} &= \left| \frac{1}{2} \langle -21/2 | \langle -21/2 | F_{1-} F_{2-} | -19/2 \rangle | -19/2 \rangle \right|^2 \sigma(k_{f2}/k_i, -1) \\ &= \left(\frac{21}{2} \right)^2 \sigma(k_{f2}/k_i, -1). \end{aligned} \quad (\text{A.17})$$

Both collisions involve identical spin states in the incoming channel so $g_d = 2$. Both collisions remove two atoms initially in $m_F = -19/2$ from the trap so $g_n = 2$.

Combining Eq. A.16, A.17, and A.1, the total collisional loss rate from the $m_F = -19/2$ state is

$$\beta_{dr}^{-19/2;-19/2} = \left\langle \left[\frac{21}{2} \left(\frac{19}{2} \right)^2 \sigma(k_{f1}/k_i, -1) + \left(\frac{21}{2} \right)^2 \sigma(k_{f2}/k_i, -1) \right] v_{\text{rel}} \right\rangle_{\text{thermal}}. \quad (\text{A.18})$$

A.3 Spin mixture, $|m_F = -17/2\rangle$ & $|m_F = -19/2\rangle$

For a spin mixture of $|-17/2\rangle$ and $|-19/2\rangle$, we must consider three different incoming two-atom spinors:

$$|\text{in}\rangle = |-17/2\rangle |-19/2\rangle, \quad \text{mixture collision;} \quad (\text{A.19})$$

$$|\text{in}\rangle = |-17/2\rangle |-17/2\rangle, \quad \text{polarized collision;} \quad (\text{A.20})$$

$$|\text{in}\rangle = |-19/2\rangle |-19/2\rangle, \quad \text{polarized collision.} \quad (\text{A.21})$$

A.3.1 $|\text{in}\rangle = |m_F = -17/2\rangle |m_F = -19/2\rangle$

For the initial state $|\text{in}\rangle = |-17/2\rangle |-19/2\rangle$, three outgoing relaxation channels exist:

$$|\text{out}\rangle = |-19/2\rangle |-19/2\rangle, \quad \text{single-spin-flip;} \quad (\text{A.22})$$

$$|\text{out}\rangle = |-17/2\rangle |-21/2\rangle, \quad \text{single-spin-flip;} \quad (\text{A.23})$$

$$|\text{out}\rangle = |-19/2\rangle |-21/2\rangle, \quad \text{double-spin-flip.} \quad (\text{A.24})$$

The cross section for channel A.22 from Eq. A.2 is

$$\begin{aligned}\sigma_{(A.22)} &= \left| \frac{1}{2} \langle -19/2 | \langle -19/2 | (F_{1z}F_{2-} + F_{1-}F_{2z}) | -17/2 \rangle | -19/2 \rangle \right|^2 \sigma(k_{f1}/k_i, -1) \\ &= \left| -\frac{1}{2} \left(\frac{19}{2} \right) \sqrt{40} \right|^2 \sigma(k_{f1}/k_i, -1) = 10 \left(\frac{19}{2} \right)^2 \sigma(k_{f1}/k_i, -1).\end{aligned}\tag{A.25}$$

The cross section for channel A.23 from Eq. A.2 is

$$\begin{aligned}\sigma_{(A.23)} &= \left| \frac{1}{2} \langle -17/2 | \langle -21/2 | (F_{1z}F_{2-} + F_{1-}F_{2z}) | -17/2 \rangle | -19/2 \rangle \right|^2 \sigma(k_{f1}/k_i, 0) \\ &= \left| -\frac{1}{\sqrt{2}} \frac{17}{2} \sqrt{\frac{21}{2}} \right|^2 \sigma(k_{f1}/k_i, 0) = \frac{21}{4} \left(\frac{17}{2} \right)^2 \sigma(k_{f1}/k_i, 0).\end{aligned}\tag{A.26}$$

The cross section for channel A.24 from Eq. A.3 is

$$\begin{aligned}\sigma_{(A.24)} &= \left| \frac{1}{2} \langle -19/2 | \langle -21/2 | F_{1-}F_{2-} | -17/2 \rangle | -19/2 \rangle \right|^2 \sigma(k_{f1}/k_i, 0) \\ &= \left| \sqrt{\frac{21}{2}} \sqrt{\frac{40}{2}} \right|^2 \sigma(k_{f2}/k_i, 0) = \left(\frac{21}{2} \right) \left(\frac{40}{2} \right) \sigma(k_{f2}/k_i, 0).\end{aligned}\tag{A.27}$$

These collisions do not involve identical spin states in the incoming channel so $g_d = 1$. These collisions remove one atoms initially in $m_F = -19/2$ and a second atom initially in $m_F = -17/2$ from the trap; therefore, there are two population collisional loss rates, both with $g_n = 1$. Combining Eq. A.25, A.26, A.27, and A.1, the total collisional loss rate from the $m_F = -19/2$ state and $m_F = -17/2$ state is

$$\begin{aligned}\beta_{dr}^{-19/2; -17/2} &= \beta_{dr}^{-17/2; -19/2} = \langle \left[10 \left(\frac{19}{2} \right)^2 \sigma(k_{f1}/k_i, -1) + \frac{21}{4} \left(\frac{17}{2} \right)^2 \sigma(k_{f1}/k_i, 0) \right. \\ &\quad \left. + \left(\frac{21}{2} \right) \left(\frac{40}{2} \right) \sigma(k_{f2}/k_i, 0) \right] v_{\text{rel}} \rangle_{\text{thermal}}.\end{aligned}\tag{A.28}$$

$$\mathbf{A.3.2} \quad |\text{in}\rangle = |m_F = -19/2\rangle |m_F = -19/2\rangle$$

See Section [A.2.2](#).

$$\mathbf{A.3.3} \quad |\text{in}\rangle = |m_F = -17/2\rangle |m_F = -17/2\rangle$$

For the initial state $|\text{in}\rangle = |-17/2\rangle |-17/2\rangle$, two outgoing relaxation channels exist:

$$|\text{out}\rangle = |-19/2\rangle |-17/2\rangle_S, \quad \text{single-spin-flip}; \quad (\text{A.29})$$

$$|\text{out}\rangle = |-19/2\rangle |-19/2\rangle, \quad \text{double-spin-flip}. \quad (\text{A.30})$$

The cross section for channel [A.29](#) from Eq. [A.2](#) is

$$\begin{aligned} \sigma_{(\text{A.29})} &= \left| \frac{1}{2} \langle -19/2 | \langle -17/2 |_S (F_{1z} F_{2-} + F_{1-} F_{2z}) | -17/2 \rangle | -17/2 \rangle \right|^2 \sigma(k_{f1}/k_i, -1) \\ &= \left| -\frac{17}{2} \sqrt{20} \right|^2 \sigma(k_{f1}/k_i, -1) = 20 \left(\frac{17}{2} \right)^2 \sigma(k_{f1}/k_i, -1). \end{aligned} \quad (\text{A.31})$$

The cross section for channel [A.30](#) from Eq. [A.3](#) is

$$\begin{aligned} \sigma_{(\text{A.30})} &= \left| \frac{1}{2} \langle -19/2 | \langle -19/2 | F_{1-} F_{2-} | -17/2 \rangle | -17/2 \rangle \right|^2 \sigma(k_{f2}/k_i, -1) \\ &= (20)^2 \sigma(k_{f2}/k_i, -1). \end{aligned} \quad (\text{A.32})$$

Both collisions involve identical spin states in the incoming channel so $g_d = 2$. Both collisions remove two atoms initially in $m_F = -17/2$ from the trap so $g_n = 2$. Combining Eq. [A.31](#), [A.32](#), and [A.1](#), the total collisional loss rate from the $m_F =$

$-17/2$ state is

$$\beta_{dr}^{-17/2;-17/2} = \langle \left[20 \left(\frac{17}{2} \right)^2 \sigma(k_{f1}/k_i, -1) + (20)^2 \sigma(k_{f2}/k_i, -1) \right] v_{\text{rel}} \rangle_{\text{thermal}}. \quad (\text{A.33})$$

Appendix B

Eigen-image analysis

This appendix contains the Python code that performs the “Eigen-image” analysis described in Ref. [34]. This implementation was written by Jack DiSciacca and Nathaniel Burdick.

The code is fairly straightforward but two modules deserve some clarification. The `levlab.image` module provides an interface to a custom data structure used in the Lev Lab through the `DyImage` object. The `DyImage.in_container()` method loads a dataset into a list-like structure. The three images (accessed as `scipy` arrays) used to generate an absorption image (see Sec. 2.2) are referenced via `DyImage.atom`, `DyImage.bright`, and `DyImage.dark`, though only the first two are used in the code below.

The function `dry_clean` in the module `levlab.laundromat` prepares an loaded `.mat` to be resaved. In particular, it removes functions that were included in the original file that `scipy.io.savemat` (loaded as `sio`) is unable to save.

```

import scipy
import pylab

import scipy.ndimage
import scipy.stats
import scipy.linalg
import scipy.io as sio
import copy
import os

import levlab.laundromat
from levlab.image import DyImage

pylab.ioff()

class Eigen(object):
    def __init__(self, ref_folder, data_folder=None, output_folder=None,
                  pixel_size=13.e-6, mag=5., x_start=50, dx=970, y_start=50, dy=400):
        '''
        Class for eigen analysis of absorption images.
        The procedure follows the outline in Chin. Opt. Lett. 5, 128-130

        ref_folder the location of reference images
        data_folder the location of data images
        output_folder the location where the images with the eigen basis generated bright
        images are written.
        pixel_size is the physical size of camera pixel (13.e-6 for PIXIS 3.75e-6 for Dragonfly)
        mag is the magnification of the imaging system
        x_start is the pixel in x direction that start the region in the image that will be processed.
        This is mainly used to avoid any artifacts near the edges of images when using Kinetics
        mode in the PIXIS.
        dx is the length in the x direction of the region of the image that is processed.
        y_start is the pixel in y direction that start the region in the image that will be processed.
        dy is the length in the y direction of the region of the image that is processed.
        '''

        self.ref_folder = ref_folder

        if data_folder is not None:
            self.data_folder = data_folder
        else:
            self.data_folder = os.path.join(self.ref_folder)

        if output_folder is not None:
            self.output_folder = output_folder
        else:
            self.output_folder = os.path.join(self.data_folder, 'eigen')

        self.pixel_size = pixel_size
        self.mag = mag
        self.x_start = x_start
        self.dx = dx
        self.y_start = y_start
        self.dy = dy

    def basis(self, file_number=25, reference_image_type='just_bright', eigen_thresh=10**8.):
        '''
        This function generates the basis set for the images.

```

```

file_number is the numbre of reference images used to create the basis,
this should be typically less that 60 images.
reference_image_type specifies what type of image file is used in the reference set
this should be 'just_bright', 'just_atom', or 'both'
eigen_thresh is the eigenvalue threshold for included an eigenvector in the basis.
this should be positive to avoid any strange eigenvectors
'''
ref = DyImage.in_container(os.path.join(self.ref_folder, '*.mat'))
ref.roi = [self.x_start, self.y_start, self.dx, self.dy]

vectors = []
average_image = scipy.zeros(ref[0].atom.size)
for img in ref[:file_number]:
    if reference_image_type == 'just_bright':
        vectors.append(img.bright.flatten())
    elif reference_image_type == 'both':
        vectors.append(img.bright.flatten())
        vectors.append(img.atom.flatten())
    elif reference_image_type == 'just_atom':
        vectors.append(img.atom.flatten())
    else:
        raise ValueError("reference_image_type must be 'just_bright', 'just_atom', or 'both'.")

    average_image += vectors[-1]*1.0

self.average_image = average_image/len(vectors)
normalized_vectors = scipy.array(vectors)-scipy.tile(average_image,(len(vectors),1))

vmatrix = scipy.matrix(normalized_vectors)

vals,vecs = scipy.linalg.eig(scipy.dot(vmatrix,vmatrix.T))

eigenvectors = [scipy.dot(vmatrix.T,vecs[:,i]) for i,val in enumerate(vals)
                 if val>eigen_thresh]

print('Number of Eigenvectors used (if less than 10, lower eigen_thresh): {0}'
      .format(len(eigenvectors)))

self.eigenvectors = scipy.squeeze(scipy.asarray(eigenvectors))

def project(self,auto_mask='false',x_center=180,atom_dx=120,y_center=700,atom_dy=160,buffer_size=10):
    '''
        This functions projects the data onto the basis. This creates a new bright image
        that is then written to file in the output_folder location.

        auto_mask is an option for masking the atoms from the absortion image.
        this can be 'true' if the fitted values in the gui are correct.
        this can be 'false' and input by hand using:
            x_center,atom_dx,y_center,atom_dy
        buffer_size is used with auto_mask = 'true' to specify the margin around the
        cloud of atoms.
    '''

    atom = DyImage.in_container(os.path.join(self.data_folder, '*.mat'))
    atom.roi = [self.x_start, self.y_start, self.dx, self.dy]

    for img in atom:
        if auto_mask == 'false':
            atom_x_start = int(x_center - atom_dx/2)
            atom_x_end = int(x_center + atom_dx/2)

```

```

    atom_y_start = int(y_center - atom_dy/2)
    atom_y_end = int(y_center + atom_dy/2)

    elif auto_mask == 'true':
        x_center = img.fitinfo.x0_global - self.x_start
        y_center = img.fitinfo.y0_global - self.y_start

        atom_dx = int(img.fitinfo.sx/self.pixel_size*self.mag*buffer_size)
        atom_dy = int(img.fitinfo.sy/self.pixel_size*self.mag*buffer_size)

        atom_x_start = int(x_center - atom_dx/2.)
        atom_x_end = int(x_center + atom_dx/2.)
        atom_y_start = int(y_center - atom_dy/2.)
        atom_y_end = int(y_center + atom_dy/2.)

    else:
        raise ValueError("auto_mask must be 'true' or 'false'.")

    mask = scipy.ones(atom[0].atom.shape)
    mask[atom_x_start:atom_x_end,atom_y_start:atom_y_end] = 0
    mask = mask.flatten()

    flat_atom = copy.copy(img.atom).flatten()
    test_atom = (flat_atom-self.average_image) * mask
    bright_image = self.average_image
    for l in self.eigenvectors:
        eigen_vect_test = (scipy.array(l)*mask)
        norm = scipy.linalg.norm(eigen_vect_test)
        #there used to be (test_atom-average_image)
        bright_image += (scipy.dot(eigen_vect_test.T, test_atom)/norm**2)*l

    bright_image = bright_image.reshape(img.atom.shape)

    print img.filename

    input_name = os.path.join(self.data_folder,img.filename)
    tempImgFile = sio.loadmat(input_name, struct_as_record=False, squeeze_me=True)
    tempImgFile['rawImage'][self.y_start:(self.y_start+self.dy),
                           self.x_start:(self.x_start+self.dx),1] = bright_image
    levlab.laundromat.dry_clean(tempImgFile)
    output_name = os.path.join(self.output_folder,img.filename)
    try:
        sio.savemat(output_name, tempImgFile)
    except IOError:
        os.mkdir(self.output_folder)
        sio.savemat(output_name,tempImgFile)

```

Appendix C

Energy levels in dysprosium

This appendix contains a diagram of the known energy levels in neutral dysprosium. An interactive version of this diagram with zooming and electronic configuration labels is available online at levlabserver2.stanford.edu/dy_levels.html. Additional interactive diagrams can be generated from any atomic spectra in Ref. [25] at levlabserver2.stanford.edu/atomic_levels/levels.cgi.

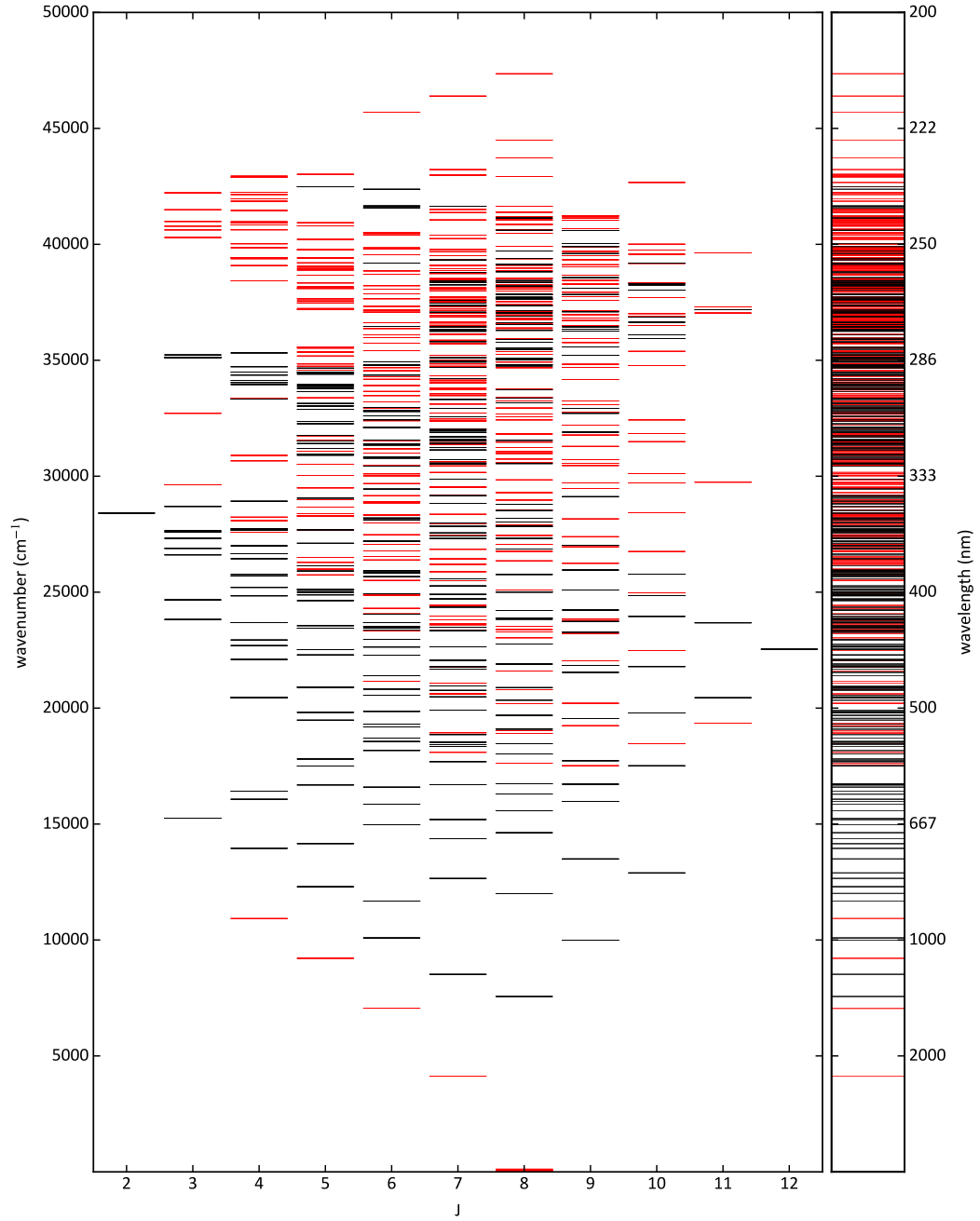


Figure C.1: Known energy spectrum of dysprosium. The total angular momentum J of each quantum state is indicated by the horizontal axis. States with odd parity are shown in black, and states with even parity are shown in red. An additional presentation that disregards the spread in J is shown on the right side to better convey the energy level density. Data from [25].

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